A GRAPHICAL SOLUTION OF THE FLOOD-ROUTING EQUATION
FOR LINEAR STORAGE-DISCHARGE RELATION

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Abstract--The problem of routing floods through reservoirs where the storage is a linear function of the outflow is treated, and a simple graphical method of obtaining the discharge from the inflow is presented.

Introduction--The authors in working on the production of synthetic unit hydrographs from basin characteristics frequently found it necessary to route time-area diagrams through various amounts of storage.

The method--In the following equations, I is the inflow in cubic feet per second or similar units; Q is the outflow in the same units; S is the storage in the reservoir and is measured in days-hours; t is the time from start of routing interval, and T is the duration of inflow in hours. It is assumed that the storage is a linear function of the outflow, that is, S = KQ. When I, Q, and S are functions of time t, the equation of continuity is I = Q + ds/dt, that is

\[ I = Q + \frac{dS}{dt} \]  

The general solution of this equation is

\[ Q = I/K \int e^{-t/K} I \, dt \]  

This equation can be integrated in cases where I is a simple function of t. For the purposes of this graphical system, it is assumed that the inflow may be taken as a block diagram, that is, the inflow is constant over short intervals.

Constant inflow--Where I is constant throughout the routing interval, the solution of (2) is

\[ Q = I + C/K \]  

where C is the constant of integration. It may be evaluated as follows: if \( Q = Q_0 \) when t is zero, then \( C/K = Q_0 - I \); whence \( Q = I + (Q_0 - I) e^{-t/K} \), or

\[ I - Q = (I - Q_0) e^{-t/K} \]  

This means that while the inflow is constant the outflow approaches the inflow logarithmically.

In Figure 1 the discharge is given by (3) between \( t = 0 \) and \( t = T \). At \( t = T \), the inflow ceases and (3) becomes the well-known equation of the recession curve for discharge from linear storage \( 0 = Q_0 e^{-t/K} \), where t is measured from the cessation of inflow. It is thus clear that \( I - Q \) between \( t = 0 \) and \( t = T \) and \( Q \) from \( t = T \) to \( t = \infty \) have the same equation and can be represented by different parts of the same curve.

If a curve, Figure 2, is cut out in celluloid or some similar material according to the equation \( 0 = Q_0 e^{-t/K} \), \( K \) being measured according to the time scale to which the inflow is plotted, this curve can be used to obtain the outflow as follows: Put the template on the inflow diagram as in Figure 3 with the axis of the curve coinciding with the value of I and slide to the right until the curve passes through the origin. Draw a pencil along the curve up to A, the time at which inflow ceases. It is clear that this represents the outflow between \( t = 0 \) and \( t = T \). Now turn the template upside down and make the time axis lie along the time axis of the diagram (because I is now zero) and slide to the right until the curve passes through A. Draw the recession curve of outflow along the template.

Inflow in block form--Very frequently the inflow may be expressed by means of a block diagram and the calculation of the outflow is simply a repetition of the method described above for a constant inflow. Having completed the first block, move the template upwards and to the right until it is in the position shown in Figure 4, that is, the time axis along the inflow value and the curve passing
through A. Then draw AB along the curve of the template. This procedure may be repeated for any number of inflow blocks until the routing is completed. Any time the inflow is less than the discharge, it will be necessary to turn the template upside down as in drawing the recession curve when I is zero.

The authors have used the method extensively for the purpose stated in the introduction and found it rapid, accurate, and easy of application.

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