A NOTE ON A INVESTIGATION INTO TWO ASPECTS OF THE RELATION BETWEEN RAINFALL AND STORM RUNOFF

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SUMMARY

(1) An empirical investigation into the relation by volume between rainfall and storm runoff in individual storms on a small catchment.
(2) The moments of the instantaneous unit hydrograph are correlated with the topographical characteristics of the catchment, and a general equation for the instantaneous unit hydrograph chosen.

RÉSUMÉ

Cette communication examine le relation, par volume, entre la précipitation et l'écoulement pendant des orages individuels.
On établit une corrélation entre les moments de l'hydrogramme unitaire instantané avec les caractéristiques topographiques et une équation générale est choisie pour l'hydrogramme unitaire.

1. VOLUME OF STORM RUNOFF AS A FUNCTION OF VOLUME OF RAINFALL

A catchment was chosen of area 21.2 km^2, on which the rainfall was measured by three continuous recording rain gauges over a period of five years. The outfall was also measured continuously over the same period. The catchment consisted of undulating farmland on London Clay with glacial sand and gravel beds. It appeared therefore to be reasonably watertight. The gravel beds stored water which provided a perennial base flow of from 0.05 to 0.15 m^3/sec. The maximum discharge recorded was of the order of 7 m^3/sec.

The record of rainfall was examined and every occurrence of 0.25 in. (0.635 cms) or more in individual storms was marked. The volume of rainfall, (expressed in inches (in.) depth over the catchment area) was taken as the average of the records of the three stations, weighted according to Thiessen's method. The volumes of storm runoff were obtained by continuing the recessions by comparison with one another, and particularly with the recession of the flood of 12th December 1954 which served as a standard storm runoff recession. The recessions were carried down to a very low base flow in order to include as much as reasonably possible of the total hydrograph in the storm runoff (see Fig. 1). The volumes of storm runoff (Q) caused by each occurrence of 0.25 in. (0.635 cms) of rainfall (R) were thus obtained, and provide, with the rainfall, the data in columns 5 and 6 of Table 1. An attempt was then made to calculate, from meteorological data, *the soil moisture deficiency* (D) at the time of occurrence of each storm. The method employed was as follows. It was assumed that the deficiency was zero whenever substantial storm runoff, more than 0.05 in. (0.127 cms), occured. (It was thus conceded that small amounts of runoff could occur due to runoff from impervious areas etc., even when a deficiency existed in the soil). The deficiency at any subsequent time was taken as the calculated evapo-transpiration plus any storm runoff less any rainfall ($D = E + q - r$) up to that time. (See Table 1, column 4).

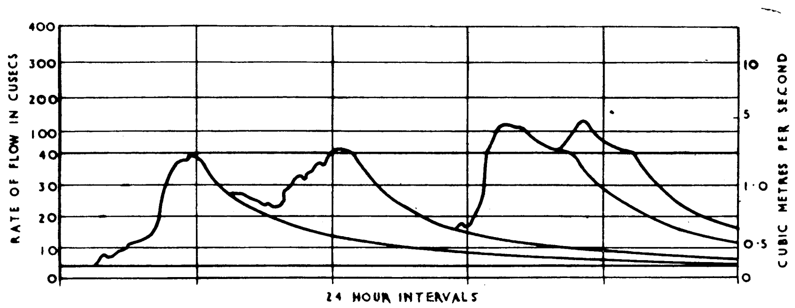

Fig. 1. A typical hydrograph showing base flow separation

The evapo-transpiration was calculated according to Penman's (1) formula, using data on humidity and temperature collected in the catchment, and wind speed and nett radiant energy as measured at Rothamsted some 30 km. away. Undoubtedly the measure of soil moisture deficiency so obtained was not sufficiently accurate, and a direct measure would be preferable. However, a direct measure was impossible as the work was being carried out on existing records.

During the summer the deficiency normally grows until it reaches several inches. We cannot say, with sufficient accuracy for this purpose, how the evapo-transpiration is affected by the deficiency, and therefore it is not possible to calculate the deficiency during the summer. On the other hand a result obtained in this investigation, (viz. that the storm runoff was negligible when the rainfall was less than the deficiency) justified the calculation of a minimum possible deficiency throughout the summer, thus providing useful data when this minimum deficiency was in excess of the rainfall. The minimum deficiency was calculated by assuming that the deficiency could never exceed 2 in. (5.08 cms) and when this level was reached the evapo-transpiration rate dropped to zero. Subsequent rainfall reduced this minimum deficiency by the amount of rainfall less the amount of any storm runoff. When the minimum deficiency so calculated exceeds the rainfall the fact is indicated in column 4, Table 1, by '>R' (greater than the rainfall). Eventually as the autumn progresses the actual deficiency falls, and so too does the calculated minimum deficiency, until a storm occurs in which the latter is less than the rainfall. While it is probable that the actual deficiency is still greater than the rainfall direct evidence is lacking, and we cannot make any statements about the relation between the deficiency and the rainfall. Such storms are marked 'U' in column 4, Table 1, indicating that the deficiency is unknown.

The choice of 2 in. as the limit of deficiency is obviously very conservative. However, the only effect of a higher figure would be to change the marking (column 4, Table 1) of some storms from 'U' to '>R'. The increase in the calculated minimum deficiency in the storms at present marked '>R' would not have any effect on the subsequent analysis.

The remaining columns of Table 1 are the duration of the storm (T) in hours and the time of the year η. The latter was defined in a manner in which it was hoped the effect of season would appear linearly. The days of the year were considered to be marked at equal angular intervals around the circumference of a circle beginning with θ = zero at the 21st March. The value of η appropriate to any day of the year is $\eta = \sin \frac{360}{365} d$, where d is the number of days since 21st March. The quantity η, is zero at the equinoxes and $+1$ and -1 respectively at the summer and winter solstices.

TABLE 1

(1) No of storm	(2) Date of storm	(3) deficiency calculated from	(4) D (inches)	(5) R (inches)	(6) Q (inches)	(7) T hours	(8) η	(9) class
1	1.12.50	29.11.50	0.03	0.54	0.19	11	— .96	a
2	3. 2.51	27. 1.51	—0.01	0.34	0.15	8	— .56	a
3	5. 2.	4. 2.	0.00	0.51	0.38	22	— .67	a
4	9. 2.	8. 2.	—0.04	0.66	0.42	21	— .62	a
5	11. 2.	9. 2.	—0.03	0.52	0.33	(12)	— .59	a
6	14. 2.	12. 2.	0.02	0.73	0.48	18	— .55	a
7	21. 2.	19. 2.	0.03	0.63	0.44	23	— .44	a
8	25. 2.	21. 2.	0.06	0.58	0.42	12	— .38	a
9	27. 2.	25. 2.	0.05	0.25	0.18	7	— .34	a
10	14. 3.	12. 3.	0.10	0.66	0.32	18	— .11	a
11	9. 4.	8. 4.	0.02	0.51	0.31	10	+ .34	a
12	9. 4.	9. 4.	0.06	0.25	0.17	4	+ .34	a
13	13. 4.	10. 4.	0.24	0.33	0.11	6	+ .41	a
14	30. 4.	13. 4.	1.52	0.25	0.00	5	+ .64	b
15	22. 5.	6. 5.	1.45	0.37	0.004	11	+ .88	b
16	27. 5.	6. 5.	1.42	0.44	0.02	14	+ .92	b
17	13. 7.	6. 5.	$<$R	0.38	0.003	5	+ .91	b
18	6. 8.	6. 5.	$<$R	0.91	0.010	11	+ .68	b
19	12. 8.	6. 5.	$<$R	0.42	0.003	10	+ .60	b
20	30. 8.	6. 5.	$<$R	0.37	0.004	4	+ .33	b
21	2. 9.	6. 5.	$<$R	0.33	0.005	5	+ .28	b
22	13. 9.	6. 5.	$<$R	0.42	0.002	4	+ .09	b
23	15. 9.	6. 5.	$<$R	0.34	0.002	5	+ .05	b
24	28. 9.	6. 5.	U	1.20	0.02	12	— .17	b
25	2.11.	6. 5.	$<$R	0.28	0.005	8	— .70	b
26	6.11.	6. 5.	U	0.69	0.02	48	— .74	b
27	11.11.	6. 5.	U	0.26	0.02	4	— .80	b
28	17.11.	6. 5.	U	0.78	0.22	18	— .86	b
29	18.11.	18.11.	0.00	0.45	0.16	17	— .87	a
30	25.11.	24.11.	0.00	0.39	0.20	6	— .92	a
31	4.12.	25.11.	—0.03	0.27	0.03	10	— .97	a
32	31. 1.52	18. 1.52	—0.38	0.26	0.14	(12)	— .73	a
33	7. 3.	19. 2.	0.20	0.40	0.12	13	— .21	a
34	7. 3.	7. 3.	0.01	0.28	0.16	8	— .21	a
35	10. 3.	8. 3.	0.06	0.49	0.28	13	— .16	a
36	23. 4.	2. 4.	0.99	0.51	0.001	13	+ .56	b
37	5. 5.	2. 4.	1.41	0.72	0.04	11	+ .71	b
38	7. 8.	12. 7.	$>$R	0.86	0.00	10	+ .67	b
39	8. 8.	12. 7.	$>$R	0.35	0.002	19	+ .66	b
40	16. 8.	12. 7.	$>$R	0.20	0.001	2	+ .55	b
41	19. 8.	12. 7.	$>$R	0.43	0.001	17	+ .52	b
42	20. 8.	12. 7.	$>$R	0.30	0.003	2	+ .49	b
43	25. 9.	12. 7.	$>$R	0.41	0.002	4	+ .41	b
44	28. 9.	12. 7.	$>$R	0.42	0.003	(12)	— .17	b
45	1.10.	12. 7.	U	0.80	0.014	14	— .23	b
46	13.10.	12. 7.	U	0.76	0.014	13	— .41	b
47	1.11.	12. 7.	U	0.47	0.015	5	— .68	b
48	3.11.	12. 7.	U	0.27	0.008	8	— .71	b
49	16.12.	13.12.	—0.18	0.39	0.21	8	— .85	a
50	19.12.	18.12.	0.00	0.35	0.14	6	— .88	a
51	12. 2.53	10. 1.53	—0.16	1.07	0.74	(12)	— .57	a

TABLE 1 (*continued*)

(1) No of storm	(2) Date of storm	(3) deficiency calculated from	(4) D (inches)	(5) R (inches)	(6) Q (inches)	(7) T hours	(8) η	(9) class
52	17. 4.53	13. 2.53	0.71	0.22	0.00	11	+ .47	b
53	27. 4.	13. 2.	1.54	0.30	0.004	2	+ .62	b
54	1. 5.	13. 2.	1.22	0.72	0.10	13	+ .66	b
55	1. 5.	1. 5.	0.06	0.41	0.12	15	+ .66	a
56	27. 5.	1. 5.	>R	0.42	0.003	8	+ .92	b
57	11. 6.	1. 5.	>R	0.42	0.005	6	+ .99	b
58	15. 6.	1. 5.	>R	0.42	0.01	6	+1.00	b
59	12. 7.	1. 5.	>R	0.63	0.002	21	+ .92	b
60	13. 7.	1. 5.	>R	0.25	0.001	13	+ .91	b
61	14. 7.	1. 5.	>R	0.32	0.004	6	+ .91	b
62	31. 7.	1. 5.	>R	0.66	0.010	12	+ .76	b
63	20. 8.	1. 5.	>R	0.41	0.004	7	+ .49	b
64	. 29. 8.	1. 5.	>R	0.34	0.002	16	+ .34	b
65	20. 9.	1. 5.	>R	0.38	0.003	5	— .04	b
66	13.10.	1. 5.	>R	1.15	0.02	8	— .41	b
67	27.10.	1. 5.	U	0.46	0.014	12	— .62	b
68	28.10.	1. 5.	U	0.36	0.014	3	— .63	b
69	30.10.	1. 5.	U	0.22	0.009	3	— .66	b
70	1.11.	1. 5.	U	0.68	0.095	14	— .67	b
71	31.12.	2.11.	—0.52	0.27	0.003	10	— .97	a
72	13. 1.54	2.11.	—1.20	0.32	0.06	3	— .91	a
73	4. 3.	19. 2.54	—0.35	0.57	0.38	17	— .26	a
74	7. 3.	4. 3.	0.10	0.32	0.12	14	— .21	a
75	31. 3.	8. 3.	0.43	0.26	0.04	7	+ .19	b
76	1. 4.	31. 3.	0.06	0.29	0.08	12	+ .21	a
77	29. 5.	3. 4.	>R	0.52	0.01	9	+ .93	b
78	8. 6.	3. 4.	>R	0.43	0.006	12	+ .98	b
79	13. 6.	3. 4.	>R	0.38	0.03	2	+1.00	b
80	17. 7.	3. 4.	>R	0.62	0.015	18	+ .89	b
81	26. 7.	3. 4.	>R	0.89	0.02	15	+ .81	b
82	6. 8.	3. 4.	>R	0.53	0.01	11	+ .68	b
83	9. 8.	3. 4.	U	0.85	0.05	7	+ .64	b
84	17. 8.	10. 8.	0.60	0.36	0.01	3	+ .53	b
85	24. 8.	18. 8.	0.83	0.62	0.05	15	+ .42	b
86	4. 9.	25. 8.	1.10	0.28	0.004	7	+ .26	b
87	13. 9.	25. 8.	1.42	0.18	0.003	7	+ .11	b
88	24.10.	25. 8.	>R	0.57	0.02	6	— .57	b
89	1.11.	25. 8.	U	0.59	0.04	18	— .68	b
90	7.11.	25. 8.	U	0.70	0.09	16	— .76	b
91	24.11.	23.11.	0.01	0.50	0.19	7	— .91	a
92	29.11.	29.11.	0.00	0.30	0.14	3	— .95	a
93	30.11.	29.11.	0.03	0.46	0.24	19	— .95	a
94	8.12.	30.11.	—0.03	0.59	0.27	12	— .98	a
95	9.12.	9.12.	—0.01	0.52	0.31	7	— .99	a
96	12.12.	10.12.	0.00	0.45	0.23	5	— .99	a

The data were next divided into two parts, (a) all storms where the rainfall was in excess of the deficiency, and (b) all the others—viz. those storms where the deficiency was known to be in excess of the rainfall and those where the deficiency was unknown. These data are shown on Figs. 2 and 3 respectively. Note that the scales of Q are different in the two figures. Because the deficiency changes rapidly from being effectively zero during the winter, to substantially more than the amount to be expected in indi-

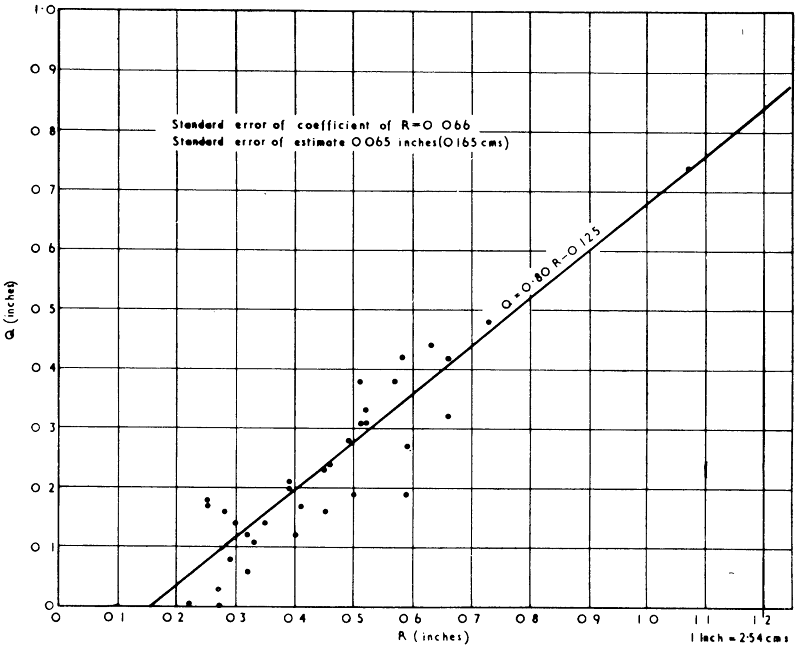

Fig. 2. — Rainfall in excess of deficiency

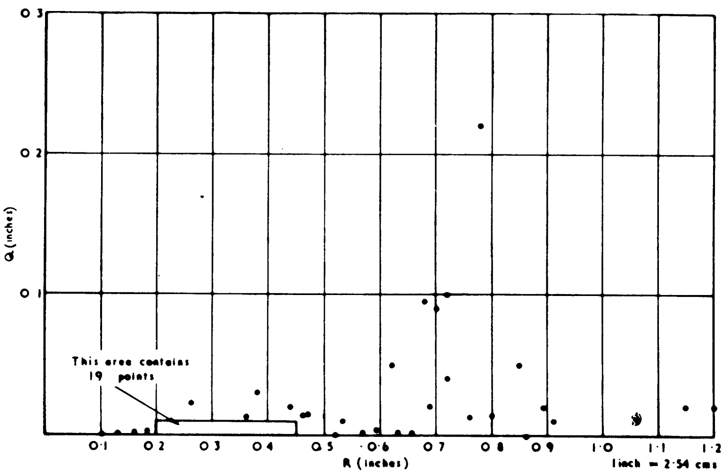

Fig. 3. — Deficiency in excess of rainfall or unknown

vidual storms during the summer, the data exhibit two rather distinct relationships, with only a very few intermediate points. The «summer» relationship (i.e., when the deficiency is greater than the rainfall) given by Fig. 2 is very simple—the runoff is negligible at least from the point of view of flood prediction. The «winter» relationship (Fig. 3) is also surprisingly simple. The data lie quite close to a straight line. One would expect the scatter from this line to be a function of the soil moisture condition and possibly the duration of the storm and time of year. Yet a linear regression analysis Table 2, fails to show this. The division of the total data into two groups on the basis of the relation between D and K emphasizes the importance of the soil moisture deficiency. It is possible that the data of Fig. 3 do not exhibit any correlation with D because within that group D may not be measured sufficiently accurately.

TABLE 2

Linear regression of Q on R, D, T, and η. Initial variance of Q = 0.0225

	Regression coefficients and their standard errors			
(1)	(2)	(3)	(4)	(5)
R	0.774 ± 0.078	0.767 ± 0.076	0.791 ± 0.067	0.799 ± 0.066
D	0.026 ± 0.050	0.035 ± 0.046	0.041 ± 0.045	
T	0.015 ± 0.024	0.017 ± 0.024		
η	0.014 ± 0.027			
Degrees of freedom	30	31	32	33
Residual variance	0.0045	0.0044	0.0043	0.0043
Standard error of estimate				0.065 inches (= 0.165 cms)
Coefficient of correlation				0.903

The prediction equation is

$$Q = 0.80\,R - 0.125 \text{ in. } (0.32 \text{ cms}) \qquad (1)$$

It is interesting to note that the rainfall R accounts for approximately 80% of the variance of Q in storms of this class—a class which can easily be distinguished except for one or two storms during the transition period. The final standard error of estimate of Q is ± 0.065 in. (0.165 cms) which is not very good, but quite understandable in view of the fact that the rainfall is normally measured at three points only and frequently at only two points.

The slope of the line of best fit in Fig. 3 is 0.80 with a standard error of ± 0.066. The 'R' axis is cut at R = 0.155 in. (0.395 cms). This suggests that an average storage of 0.155 in. must first be made up before runoff occurs and 80% of the remainder of the rainfall contributes to storm runoff and 20% to base flow.

There can be little doubt that a more direct measure of soil moisture deficiency would improve the correlation and it would seem that measurement of this quantity would help considerably in flood forecasting.

2. The relation between the unit hydrograph and the catchment characteristics

A fuller account of this work is available in ref. (2).

The object of this part of the investigation was to find empirically a general equation for the instantaneous unit hydrograph (IUH) and to find correlations between parameters of this equation and the catchment characteristics.

2.1. *Unit Hydrograph Parameters*

Let the equation of the IUH be $u = u(t)$ i.e. the storm run-off due to a volume V of effective rainfall is

$$q(t) = V.u(t) \tag{2}$$

The nth moment of the IUH about the instant of effective rainfall is

$$U'_n = \int_0^\infty t^n \ u(t)\,dt \Big/ \int_0^\infty u(t)\,dt \tag{3}$$

Let the corresponding moments about the centre of area of the IUH be U_n. Then a set of IUH parameters, which does not depend on the finally chosen general IUH equation, is

$$\left.\begin{aligned} m_1 &= U'_1 && a \\ m_2 &= U_2/(U'_1)^2 && b \\ m_3 &= U_3/(U'_1)^3 && c \\ m_n &= U_n/U'_1)^n && d \end{aligned}\right\} \tag{4}$$

Obviously m_1 is a measure of the lag of the IUH and m_2 and m_3 are measures of the spread and skew respectively. All the m's except m_1 are dimensionless. An important convenience attached to using moments as IUH parameters is that the values of the moments may be found, even in a complex flood, from the corresponding moments of the effective rainfall and storm runoff, without actually deriving the IUH. If the nth moments of the effective rainfall, storm runoff and IUH about the origin are defined as I'_n, Q'_n, U'_n, respectively, and the corresponding moments about the respective centres of area are defined as I'_n, S'_n, U'_n respectively, the following equations can be shown to relate the moments.

$$\left.\begin{aligned} U'_1 &= Q'_1 - I'_1 && a \\ U_2 &= Q_2 - I_2 && b \\ U_3 &= Q_3 - I_3 && c \\ U_4 &= Q_4 - 61_2U_2 - I_4 && d \end{aligned}\right\} \tag{5}$$

Proofs of these equations are given in reference (2) and (3).

By definition

$$\left.\begin{aligned} m_1 &= U_1 && a \\ m_2 &= U_2/(U'_1)^2 && b \\ m_3 &= U_3/(U'_1)^3 && c \end{aligned}\right\} \tag{6}$$

2.2. *The Correlations*

Values of the U's, and thence the m's were obtained for 26 catchments throughout Great Britain. The Catchments varied in area from 12.5 km² to 2230 km², in main channel length from 7.1 km to 134 km, in channel slope from 6.8 in 10,000 (ten thousand) to 538 in 10,000, and in overland slope from 150 in 10,000 to 3030 in 10,000.

The determination of the moments required the prior determination of the storm runoff—base flow separation. This was accomplished by an arbitrary but consistent method.

The duration of storm runoff was taken as being three times m_1 thus providing a point on the recession at which storm runoff was assumed to cease. This point was joined by a straight line to the point on the hydrograph at which the rise began. The factor of three was arbitrarily chosen because in the majority of cases it gave a separation which looked reasonable. In determining m_1 a method of trial and error was necessary. A point was chosen, the separation made and m_1 calculated. If the duration of storm runoff was not $3m_1$ a second trial was made. Generally a 2nd trial only was necessary.

Linear regression analyses were applied to the relation between log m_1 and log m_2 as dependent variables, and the logs of the following catchment characteristics as independent variables.

A catchment area in km^2

L length, in km, of the main stream from the gauging site to the catchment boundary.

S_1 a measure of the slope of the main channel (expressed in parts per 10,000) i.e. if slope is n parts per 10,000 $S_1 = n$ See Fig. 4.

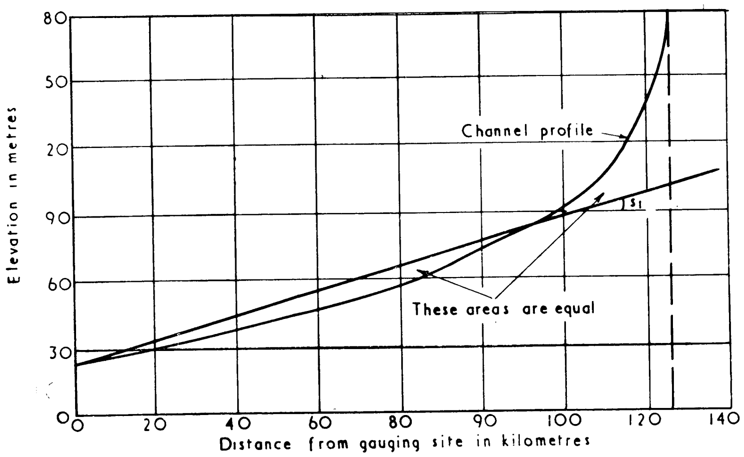

Fig. 4. — Main channel slope

S_2 A measure of the overland slope, defined as the mean of the slopes measured at the intersections of a grid imposed on a map of the catchment. The slope at each grid point was obtained by measuring the shortest distance through the point, between two contours at 25 feet (8.2 m) intervals. This slope is also expressed in parts per 10,000.

Other topographical characteristics were measured, but significant independent correlations were obtained only with those listed.

2.3. *Equations for m_1*

Equations 7 and 8 were found to give the best determinations of m_1

$$m_1 = 20.7\, A^{0.3}\, S_2^{-0.3} \qquad (7)$$

initial variance of log m_1 = 0.125
residual variance for log m_1 = 0.024
corresponding to a standard error of estimate of a factor 1.43
coefficient of correlation 0.90
standard error of index of A = ± 0.023
standard error of index of S_2 = ± 0.044

$$m_1 = 17.3\, L^{0.3}\, S_1^{-0.33} \tag{8}$$

initial variance of log m_1 = 0.125
residual variance of log m_1 = 0.025
corresponding to a standard error of estimate of a factor 1.44
coefficient of correlation 0.90
standard error of index of L = ± 0.059
standard error of index of S_1 = ± 0.046

2.4. *Equations for m_2*

Here we may distinguish two cases (a) where m_1 is known, and (b) where m_1 is known only through equation 7 and 8. In the first case the best equation for m_2 is :

$$m_2 = 1.0\, m_1^{-0.2}\, S_2^{-0.1} \tag{9}$$

Initial variance of log m_2 = 0.0140
residual variance of log m_2 = 0.0109
corresponding to a standard error of estimate of a factor 1.26
coefficient of correlation = 0.51
standard error of index of m_1 = ± 0.047
standard error of index of S_2 = ± 0.033

In the second case

$$m_2 = 0.43 L^{-0.1} \tag{10}$$

Initial variance of log m_2 = 0.0140
residual variance of log m_2 = 0.0112
corresponding to a standard error of estimate of a factor 1.28
coefficient of correlation 0.45
standard error of index of L = ± 0.031.

2.5. *The IUH Equation*

As the correlation of m_2 with the catchment characteristics was not well defined no attempt was made to correlate m_3 with the characteristics. Instead a general two parameter equation (or method of generation) was sought for the IUH. The choice between several two parameter forms was made by the criterion that that form was best which gave the closest agreement between the third moments of the data and the equation, when the two parameters of the IUH equation were determined by equating 1st and 2nd moments of the data and the equation. For this purpose a plot of $m_3 : m_2$ was made (Fig. 5) and the lines corresponding to several possible two parameter forms were drawn through the scatter. These lines are not shown in fig. 5. The equation whose $m_2 : m_3$ line best represented the scatter was chosen as the best general IUH equation. It is clear from fig. 5 that the line corresponding to eq. 11 was as good a fit as could be expected.

$$u(t) = \frac{1}{K\,\Gamma(n)}\, e^{-t/K}\, (t/K)^{n-1} \tag{11}$$

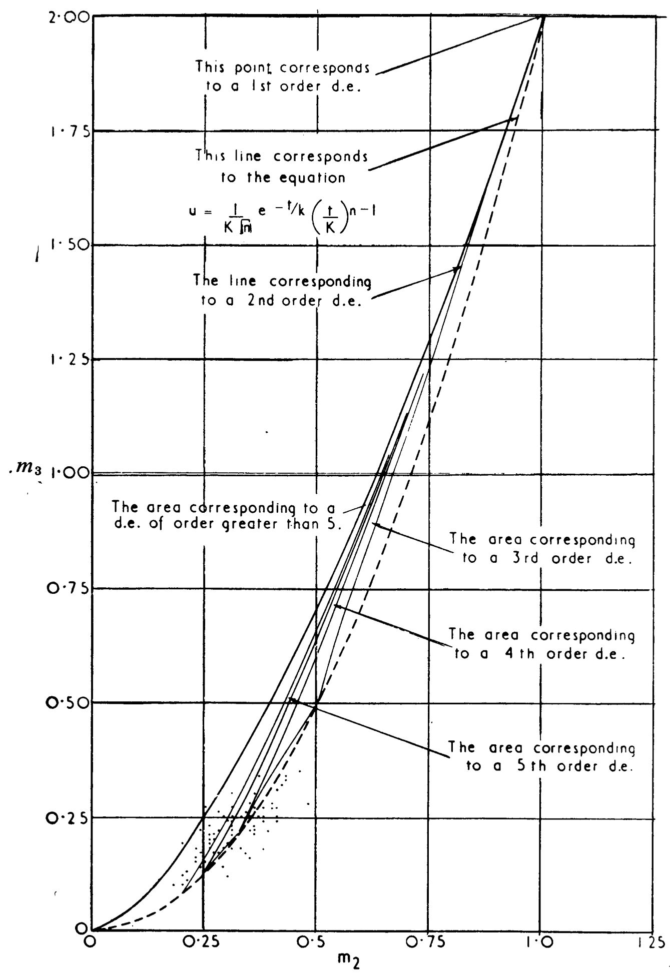

Fig. 5. — Moments of the pulse responses for differential equations of various orders

The moments of this equation are very simple, being :

$$\left.\begin{aligned} U_1' &= nK \\ U_2 &= nK^2 \\ U_3 &= 2nK^3 \end{aligned}\right\} \qquad (12)$$

and hence

$$\left.\begin{aligned} m_1 &= nK \\ m_2 &= 1/n \\ m_3 &= 2/n^2 \end{aligned}\right\} \qquad (13)$$

or

$$\left.\begin{aligned} nk &= m_1 \\ n &= 1/m_2 \end{aligned}\right\} \qquad (14)$$

Equation 11, together with equation 7 or 8 and equation 10, can be used to obtain estimates of the parameters n and K for a catchment where no records exist. These parameters can be used to define the IUH. The equation of the unit hydrograph of finite duration T corresponding to equation 11 is :

$$u(\mathrm{T},t) = \frac{1}{\mathrm{T}}\left[\mathrm{I}\,(n,t/\mathrm{K}) - \mathrm{I}\,(n, \frac{t-\mathrm{T}}{\mathrm{K}})\right] \tag{15}$$

where I $(n,\ t/\mathrm{K})$ is the value of the incomplete gamma function of order n at t/K. Equation 15 is the suggested general equation of the unit hydrograph when the duration of effective rainfall is T.

2.6. *Physical analogy*

A simplified model of a catchment might consist of a series of reservoirs such that the first discharges into the second and so on, the outfall of each so controlled that the storage in any one is K times the discharge from it. Such a catchment has an exactly linear response and therefore an exact IUH. It can be shown (2) (4) that the equation of this IUH is of the form of eq. 11.

2.7. *The general differential operator with constant coefficients*

Let the linear relation between the input (the effective rainfall $i\ (t)$) and the output (the storm runoff $q\ (t)$) be defined by a general linear differential equation with constant coefficients, restricted only by the requirements of stability and high damping.

$$q(t) = \frac{1}{\mathrm{F(D)}}\, i(t)$$

where F(D) is a general polynomial in the differential operator D = d/dx, limited only in that its roots are real and positive. The corresponding pulse response can be shown (2) to have moments such that the point $m_2 : m_3$ must lie with in the loop of fig.5. If the order of the differential equation is limited the point $m_2 : m_3$ is further restricted as shown in fig. 5. It is clear from fig. 5 that if the loop were a single line principle the restriction on the general principal of superposition, which is the basic definition of a linear relation and consequently of the unit hydrograph idea, implied by the assumption of such a linear differential equation between $q(t)$ and $i(t)$, would be no less than the restriction imposed by assuming equation 11 as the general equation of the pulse response. Conversely the assumption that the general IUH equation is equation 11 is only a little more restrictive than the assumption that the output is obtained by a linear differential equation with constant coefficients operating on the input.

In many branches of applied science the latter assumption is often used as the practical equivalent of the superposition principal when the system is stable and highly damped.

It may therefore be suggested that the assumption of an IUH of the form of equation 11 is not likely to be unduly restrictive.

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