DISCUSSION

with the extreme-value law for \( C_v = 0.364 \). In other words, when \( C_v = 0.364, KL = KQ \) and (14) gives \( R_t = 1 \). This is not shown in Figure 3, but can be readily verified by the above method or by the author's nomograms.

Note that when an equation like (14) is used, the sample for the log-probability frequency analysis is the same sample for the extreme-value frequency analysis, and hence the period of record in both cases is equal to the same value \( n \). The value of \( X_r \) as represented in (11) is based on a theoretical distribution of \( n = \infty \). For a sample of limited size, however, \( X_r \) is also a function of \( n \) just as \( X_{T0} \) is a function of \( n \). Since \( X_{T0} \) is made a function of \( n \) through the use of a plotting position formula containing \( n \), it is possible that \( X_r \) is made a function of \( n \) by a similar procedure. Otherwise, a procedure developed by Foster [1955, Fig. A] is applicable for the development of \( X_r \) as a function of \( n \). It seems to the writer that (12) is valid only if \( n = \infty \) or if a correction for \( n \) is to be applied to \( X_r \) in (11) in the course of the derivation of (12).

REFERENCES


L. L. WEISS (U. S. Weather Bureau, Washington, D. C.; Author's closure)—I want to thank Chow for his discussion. It is gratifying to see the verification of my examples by his general method for frequency analysis. Since the use of his table of frequency factors or my nomogram give the same results, the choice is a matter of user preference.

With regard to Chow's interesting remarks in his concluding paragraph, I would leave moot the question of the dependence of the \( X_r \) on sample size. The comparison which I wanted to illustrate in Figure 3 was that between the values when computed in the usual fashion using sample mean and standard deviation. Nothing was said concerning the confidence bands for either method. If these are taken into account, we may say that for practical applications the differences between the values determined by the two methods are not important.

Discussion of “Frequency of Discharges from Ungaged Catchments”

BY J. E. NASH

[Trans., 37, 719-725, 1956]
TABLE 2 - Computation of the frequency of peak discharge

<table>
<thead>
<tr>
<th>T (hr)</th>
<th>P (cm)</th>
<th>CR (cm)</th>
<th>R (cm)</th>
<th>P2 (cm)</th>
<th>P1 (cm)</th>
<th>P2 - P1 (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1.17</td>
<td>0.427</td>
<td>2.20</td>
<td>0.09</td>
<td>-</td>
<td>0.009</td>
</tr>
<tr>
<td>1.0</td>
<td>1.133</td>
<td>0.443</td>
<td>2.20</td>
<td>0.030</td>
<td>0.009</td>
<td>0.021</td>
</tr>
<tr>
<td>1.5</td>
<td>1.110</td>
<td>0.450</td>
<td>2.23</td>
<td>0.060</td>
<td>0.028</td>
<td>0.032</td>
</tr>
<tr>
<td>2.0</td>
<td>1.060</td>
<td>0.471</td>
<td>2.25</td>
<td>0.092</td>
<td>0.057</td>
<td>0.035</td>
</tr>
<tr>
<td>2.5</td>
<td>1.000</td>
<td>0.500</td>
<td>2.35</td>
<td>0.110</td>
<td>0.080</td>
<td>0.030</td>
</tr>
<tr>
<td>3.0</td>
<td>0.925</td>
<td>0.541</td>
<td>2.40</td>
<td>0.130</td>
<td>0.100</td>
<td>0.030</td>
</tr>
<tr>
<td>3.5</td>
<td>0.870</td>
<td>0.575</td>
<td>2.45</td>
<td>0.145</td>
<td>0.120</td>
<td>0.025</td>
</tr>
<tr>
<td>4.0</td>
<td>0.805</td>
<td>0.622</td>
<td>2.55</td>
<td>0.145</td>
<td>0.125</td>
<td>0.020</td>
</tr>
<tr>
<td>4.5</td>
<td>0.752</td>
<td>0.665</td>
<td>2.65</td>
<td>0.140</td>
<td>0.125</td>
<td>0.015</td>
</tr>
<tr>
<td>5.0</td>
<td>0.685</td>
<td>0.730</td>
<td>2.75</td>
<td>0.130</td>
<td>0.115</td>
<td>0.015</td>
</tr>
<tr>
<td>5.5</td>
<td>0.621</td>
<td>0.805</td>
<td>2.90</td>
<td>0.115</td>
<td>0.105</td>
<td>0.010</td>
</tr>
<tr>
<td>6.0</td>
<td>0.571</td>
<td>0.875</td>
<td>3.00</td>
<td>0.100</td>
<td>0.090</td>
<td>0.010</td>
</tr>
<tr>
<td>6.5</td>
<td>0.527</td>
<td>0.950</td>
<td>3.10</td>
<td>0.090</td>
<td>0.085</td>
<td>0.005</td>
</tr>
<tr>
<td>7.0</td>
<td>0.490</td>
<td>1.020</td>
<td>3.23</td>
<td>0.080</td>
<td>0.075</td>
<td>0.005</td>
</tr>
</tbody>
</table>

\[ \Sigma = 0.262 \]

duration-frequency curves at the location of the watershed are shown in Figure 8. Columns 5 and 6 in Table 2 give the frequencies \( F_2 \) and \( F_1 \) respectively at the end \( (T + h) \) and beginning \( (T - h) \) of the duration interval \( \Delta T \). These frequencies are obtained from the curves in Figure 8. For \( T = 1.5 \) hr, as an example, the frequencies \( F_2 = 0.060 \) and \( F_1 = 0.028 \). The dashed line in Figure 8 shows how these frequencies for various durations are found. Column 7 in Table 2 gives the difference \( F_2 - F_1 \) or \( F(R, T, O) \) as defined by (5). Consequently, according to (7), the sum of all values in column 7 should be equal to the frequency of 500 cfs as the peak discharge. This is found to be 0.262 or 3.8 yr.

Taking the rainfall depth \( R \) as a parameter, the curves in Figure 8 can be replotted with the frequency \( F \) against the duration \( T \) as shown in Figure 9. Also in Figure 9 are plotted the values

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**Fig. 8 - Rainfall depth-duration-frequency curves**
from Table 2 for \( T \pm h \) against \( F_2 \) and \( F_1 \). For each duration interval, or 0.5 hr in the example, a right triangle shown in dashed line can be constructed. In this triangle, the vertical side represents \( F_2 - F_1 \), the horizontal side represents the duration interval \( \Delta T = 0.5 \) hr, and the hypotenuse has a slope equal to the slope of the solid line representing the rainfall depth \( R \) corresponding to the duration \( T \). According to (7), the frequency of the peak discharge is equal to the sum of the lengths of all vertical sides of the triangles; that is

\[
F_q = \sum_{0}^{T} (F_2 - F_1) \tag{17}
\]

or, expressed in the integral form

\[
F_q = \int dF = \int (dF/dT) dT \tag{18}
\]

The term \( dF/dT \) represents the slope of the solid line in Figure 9. Taking the rainfall depth \( R \) as a parameter, this term \( dF/dT \) can be plotted against the duration \( T \) as shown in Figure 10.

Plot the duration \( T \) in column 1 of Table 2 against the rainfall \( R \) in column 4, resulting in a dashed curve in Figure 10. According to (18), it is apparent that the area under this curve should be equal to the frequency of the peak discharge. This area can be planimetered and is found to be 0.280 or 3.6 yr, which agrees satisfactorily with the result obtained by the numerical procedure in Table 2.

In comparing the numerical procedure with the graphical procedure, it is obvious that the accuracy of the former depends on the magnitude of the duration interval \( \Delta T \) or \( 2h \), whereas the accuracy of the latter depends much on the scale used for constructing the curves.
DISCUSSION

The writer believes that the Dillon data used in author's analysis tends to overestimate the frequency. In the analysis of the data, Dillon [1954] has followed a conventional concept that "any one storm can allow an entry for each time interval or duration, and any one portion of the storm-trace can be included in one or more of the duration intervals." For example, the amounts of rainfall of July 23, 1916 occurring in given durations (or less) as listed by Dillon are:

<table>
<thead>
<tr>
<th>Duration (min)</th>
<th>10</th>
<th>15</th>
<th>30</th>
<th>45</th>
<th>60</th>
<th>90</th>
<th>120 etc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount (inch)</td>
<td>0.50</td>
<td>0.65</td>
<td>0.91</td>
<td>0.94</td>
<td>0.95</td>
<td>0.96</td>
<td>0.98 etc.</td>
</tr>
</tbody>
</table>

The first value 0.50 was used in the analysis for $T = 10$ min or less, the second value of the same storm 0.65 was used in the analysis for $T = 15$ min or less, and so on. Thus, the effect of this storm has been included in the analyses of several durations; whereas the storm actually occurred only once and, in accordance with the implication of the author's method, it could produce only one independent discharge. In other words, this storm has been counted several times in the author's analysis and hence the resulting frequencies have been overestimated. If the actual duration of this storm is known, the storm should be counted or included in the analysis only once for this duration only. The curves as shown in Figure 8 were derived from an analysis based on this new concept. A comparison of these curves with the conventional curves indicates that the overestimation of the latter exists for durations less than 1.5 hr in the given example. For durations greater than 1.5 hr the overestimation is practically negligible.

It is noted that the assumption of uniformity of rainfall intensity contained in (2) is an error which fortunately tends to underestimate the result and thus serves to compensate somewhat the overestimated effect as described in the preceding paragraph.

Assumption (1) in the paper accepts a constant runoff coefficient in the analysis. However, as mentioned in the beginning of this discussion, the runoff coefficient depends not only on the major factors such as the soil type, antecedent watershed conditions, and land use, but also on the magnitude, duration, and distribution of the individual storms. In fact, the runoff coefficient is a variable instead of a constant. It has been the writer's belief [Chow, 1956] that the runoff from a watershed is a product in the hydrologic cycle that is affected by two major groups of factors: climatic factors and the physiographic factors. In accordance with this concept, the frequency of the discharge may be conceived as the product of the frequencies of the rainfall and the runoff condition. It seems that the method for the determination of discharge frequency would be more satisfactory if the runoff coefficient could be treated also as a statistical variable. Unfortunately, the frequency analysis of runoff coefficient is a subject which has not yet been fully investigated.

REFERENCES


J. P. Farrell (Office of Public Works, Engineers Branch, Dublin, Ireland, received May 2, 1957)—This paper fills a serious gap in the literature on the problem of relating runoff frequencies to rainfall frequencies. However the method set out by the author of applying the principle that "On a given catchment the frequency of $q$ is the sum of the frequencies of all storms which cause $q$" where $q$ is the peak discharge in cfs/mi² is somewhat complex and mistakes could easily creep in. The following alternative derivation and method of working may therefore be of interest.

As described by the author the ordinate at time $t$ of the unit hydrograph for effective rainfall duration $T$ may be derived from the instantaneous unit
hydrograph and is

\[ U(T, t) = \frac{1}{T} \int_{t-T}^{t} U(0, \tau) \, d\tau \]

By choosing \( t \) so as to maximize this integral the peak of the unit hydrograph of period \( T \) is obtained. This latter peak may be denoted by \( P_T \) and the peak of the instantaneous unit hydrograph by \( P_0 \). A curve may then be plotted of \( P_0/P_T \) against \( T \) or to get ratios on both axes, of \( P_0/P_T \) against \( T/t_P \) where \( t_P \) is the time to peak of the instantaneous unit hydrograph. Such a curve derived from Commons basic hydrograph which the author used as his instantaneous unit hydrograph shape is shown by the full line in Figure 11. Curves derived in the same way from other well known unit graphs are shown for comparison. If \( P_0 \) is the peak in cfs/mi² due to one inch instantaneously on the catchment, then ordinates of the curve give for any duration \( T \) of rainfall, the amount of rainfall in inches of that duration necessary to produce a peak equal to the instantaneous peak \( P_0 \). In other words, the curve shows all the rainfall combinations which could produce such peak. A plot of all the combinations to produce some other peak \( rP_0 \) will be got by multiplying all the \( P_0/P_T \) ordinates by \( r \). According to Dillon's rainfall frequency formula used by the author the frequency of occurrence in times per year of \( R \) inches in \( T \) or less hours is equal to 0.036 \( T^3/R^4 \). This implies that the frequency of the \((R, T)\) combination is measured by the first derivative 0.072 \( T/R^4 \) \( dT \) which is a function of position along the curve described above. It is the writer's suggestion that the frequency of \( q \) may be taken as the line integral of the function 0.072 \( T/R^4 \) along the curve showing all the rainfall combinations which could produce \( q \), that is

\[ F_q = \int 0.072 T/R^4 \, dT \]

Modified by the introduction of term \( \mu \) used by the author to allow for the diminution in rainfall frequency with increasing area, the expression for the frequency of \( q \) becomes

\[ F_q = \int 0.072\mu T/R^4 \, dT \]

![Fig. 11 - Depth-duration curves of rainfall to produce a peak equal to the instantaneous unit hydrograph peak from different unit graphs](image)
Values of $T/R^4$ taken from the curve can be calculated, multiplied by the appropriate values of $\mu$ listed by the author and plotted against $T$. The area between this last curve and the $T$ axis then gives the required frequency. Since the rainfall of duration $T$ to produce $q$ is $R = q/P_T$, the rainfall frequency function $0.072 T/R^4$ can be written in the form

$$0.072 \left( \frac{T}{t_{P_0}} \right) \left( \frac{P_0}{P} \right) \left( \frac{P}{q} \right)^{-5}$$

Once a rainfall frequency formula (such as Dillon's) has been selected, the quantity in the first large bracket above is purely a function of the instantaneous unitgraph shape being used and need be calculated only once. In the case of Commons' basic hydrograph 1200 volume units represent 645 ft/hr/sec and 60 discharge units represent $P_0$ cfs also $t_{P_0}$ equals 14 time units. Therefore $t_{P_0} = 451.5/P_0$ hours.

To exemplify the suggested method, the particular case of $P_0 = 47$ cfs/mi$^2$ corresponding to an area of 50 mi$^2$ and slope of 200 pct is worked out in Table 3 for a $q$ of 74.7 cfs/mi$^2$. The frequency of other $q$'s will of course be inversely proportional to $q^4$. Columns (1) and (2) in Table 3 are a tabulation of Figure 11. Column (3) derived from them is a function only of Commons' hydrograph and the rainfall frequency formula. Column (4) is the product of values in Column (3) by 3.21 which is the value of $[t_{P_0}/P_0(q)]^5$ in this case. To insert values of $\mu$ it is necessary to obtain absolute values of $T$ from column (1) by multiplying the latter by 9.60 hr the value of $t_{P_0}$ on this catchment. The final column consists of the products of columns (4) and (6) and is plotted as the full line in Figure 12 where it is described as $F(R, T, A)$. $P_0$ is the area under this curve and amounts to 0.448 times/yr which is reasonably close to the value given by the author's chart. The curve which would be obtained by omission of the $\mu$ factor is plotted as a dotted line on the same diagram to the scale marked $F(R, T, O)$. Its very great effect can there be seen and it is most unfortunate that there is such a dearth of data of this type. Figure 11 demonstrates that there is practically no difference between these curves when derived from different unitgraphs. Hence the frequencies obtained with these unitgraphs will be proportional to their $t_{P_0}$ values.

J. E. Nash (Department of Scientific and Industrial Research, Hydraulics Research Station, Howbery Park, Wallingford, Berks, England; Author's closure)—The purpose of this paper was to draw attention to the fact that once one departed from Kuichling's assumption that the peak discharge corresponding to a given frequency is directly proportional to the amount of rain in a certain critical period, then very great difficulties arose in relating discharge frequency to rainfall frequency. The method proposed assumed as basic data (1) a unit hydrograph correlation, (2) a rainfall formula giving the frequency with which any amount of rain may be expected in any given time or less, and (3) a formula showing the relation between the average rainfall over an area and the rainfall at a point for a given frequency and duration.

Both Chow and Farrell have shown how the author's numerical calculation could be simplified by the use of semi-graphical procedures. Farrell has also shown clearly that the shape of the basic unit hydrograph is not critical and he has pointed to the importance of the relation between the amount of rainfall over a catchment and the amount at a point. In this connection the data used by the author, which are the only such data available for the British Isles, are very unsuitable. It is clear that very much progress cannot be made until reliable data on this point become available.
Chow has pointed to the overestimate involved in using Dillon’s rainfall formula. He emphasizes the fact that Dillon’s method of analysis, which is the standard one, permitted more than one entry for one storm, and the storm of July 23, 1916 provided a whole series of entries while, of course, providing in fact only one peak discharge. This is undoubtedly true but it is very easy to overestimate the effect of this multiple use of one storm. The storm of July 23, 1916 provided say seven entries but each of these entries corresponds to a different discharge. It does not follow, as one would perhaps conclude at first sight, that a certain discharge is overestimated six times. An example will make this clear.

Consider a catchment which is very small and very steep so much so that $U(O, P) = 300$ cfs/mi$^2$. This is indeed an extreme case. Suppose we are counting the occurrences of a discharge equal to or greater than 112 cfs/mi$^2$ and we used a runoff coefficient of 0.5 we therefore require 0.75 inch of rain instantaneously, or any of the following amounts in corresponding times:

<table>
<thead>
<tr>
<th>Durations (min)</th>
<th>10</th>
<th>15</th>
<th>30</th>
<th>45</th>
<th>60</th>
<th>90</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount (inch)</td>
<td>0.75</td>
<td>0.76</td>
<td>0.76</td>
<td>0.77</td>
<td>0.80</td>
<td>0.83</td>
<td>0.91</td>
</tr>
</tbody>
</table>

Our analysis is equivalent to approaching each storm in the record and adding up (1) the number of occurrences of 0.91 inch, or more, in 120 min less the number of occurrences of 0.91 inch or more in 90 min, namely, $F(R, T + h, A) - F(R, T - h, A)$ plus (2) the number of occurrences of 0.83 inch, or more, in 90 min less the number of occurrences of 0.83 inch, or more, in 60 min plus (3) the number of occurrences of 0.80 inch, or more, in 60 min less the number of occurrences of 0.80 inch, or more, in 45 min plus (4) the number of occurrences of 0.77 inch, or more, in 45 min less the number of occurrences of 0.77 inch, or more, in 30 min plus (5) the number of occurrences of 0.76 inch, or more, in 30 min less the number of occurrences of 0.76 inch, or more, in 15 min plus (6) the number of occurrences of 0.76 inch, or more, in 15 min less the number of occurrences of 0.76 inch, or more, in 10 min.

Now (1)(2)(3)(4) furnish one minus one occurrence each, (5) furnishes one minus zero occurrence, (6) (and any subsequent) furnishes zero minus zero occurrences. Consequently our analysis takes only one occurrence equal to or greater than 112 cfs/mi$^2$ from this storm on a catchment of $U(O, P) = 300$ cfs/mi$^2$.

If we consider the number of occurrences of other discharges provided by this storm we shall always find that not more than one period will provide one minus zero occurrence. All the other periods will provide either one minus one or zero minus zero occurrences. The period which provides one minus zero occurrences will vary depending on the magnitude of the discharge whose frequency is being investigated, the higher the discharge the longer the period which proves critical.

For any other catchment where $U(O, P)$ is not greater than 300 cfs/mi$^2$ a similar result will be obtained. If $U(O, P)$ is taken very much greater than this it is possible that more than one occurrence will be registered. However such a catchment would be entirely outside the range visualized in this paper. It is possible however that a storm of greater duration than that of the July 23, 1916 would register two or more occurrences of a chosen discharge, even on a more moderate catchment and we must therefore accept the possibility of some overestimation. It would appear however to be very much less than would at first sight be expected.

There appears to be no way of eliminating this difficulty except by a radically different approach to the rainfall analysis. Chow mentions an analysis in which any storm is allowed to produce one entry only (namely, the amount of the storm in the total duration of the storm). This however raises further difficulties. Consider a rain storm containing a short break in its center, such a storm might produce two distinct peak discharges on a small catchment and possibly only one peak discharge on a large catchment. It would be desirable therefore, when preparing the rainfall frequency formula, to consider this storm as two storms when considering small catchments and as one storm when considering large catchments. This would require a different analysis for each different catchment.