

Interpretation of Earth Tide Response of Three Deep, Confined Aquifers

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The response of a confined, areally infinite aquifer to external loads imposed by earth tides is examined. Because the gravitational influence of celestial objects occurs over large areas of the earth, the confined aquifer is assumed to respond in an undrained fashion. Since undrained response is controlled by water compressibility, earth tide response can be directly used only to evaluate porous medium compressibility if porosity is known. Moreover, since specific storage S_s quantifies a drained behavior of the porous medium, one cannot directly estimate S_s from earth tide response. Except for the fact that barometric changes act both on the water surface in the well and on the aquifer as a whole while stress changes associated with earth tides act only in the aquifer, the two phenomena influence the confined aquifer in much the same way. In other words, barometric response contains only as much information on the elastic properties of the aquifer as the earth tide response does. Factors such as well bore storage, aquifer transmissivity, and storage coefficient contribute to time lag and damping of the aquifer response as observed in the well. Analysis shows that the observation of fluid pressure changes alone, without concurrent measurement of external stress changes, is insufficient to interpret uniquely earth tide response. In the present work, change in external stress is estimated from dilatation by assuming a reasonable value for bulk modulus. Earth tide response of geothermal aquifers from Marysville, Montana; East Mesa, California; and Raft River Valley, Idaho, were analyzed, and the ratio of S_s to porosity was estimated. Comparison of these estimates with independent pumping tests shows reasonable agreement.

INTRODUCTION

The response of water levels in wells penetrating confined aquifers to the earth tides has been known for over 100 years. Since the magnitude response is related to the elastic characteristics of the aquifer, hydrogeologists, geophysicists, and other earth scientists are interested in interpreting this response in terms of the elastic parameters of the aquifer. This method of parameter estimation is especially interesting because the experiment is a passive one which provides an average value on a large scale not ordinarily attainable through conventional experiments.

Many attempts have been made in the literature to interpret earth tide response data. Melchior [1960], following the conceptualization of Blanchard and Byerlee [1935], treated the aquifer as a cavity and derived an expression for estimating fluid pressure changes due to earth tides. As pointed out by Bredehoeft [1967], Melchior's expression grossly underestimates the magnitude of aquifer response. Bredehoeft [1967] carried out an extensive survey of the literature and proceeded to attempt to estimate the specific storage coefficient S_s of the aquifer from earth tide response. He also showed that Melchior's idealization of the aquifer as a cavity neglected the important phenomenon of matrix compressibility, thus leading to an underestimation of aquifer response. As will be seen later, Bredehoeft's analysis is based on a critical assumption which may be questioned. Bodvarsson [1970] idealized the homogeneous permeable rock as a volume of spherical or nonspherical geometry embedded in a formation of unspecified permeability and suggested that S_s could be obtained. Following a similar approach, A. G. Johnson and A. M. Nur (unpublished manuscript, 1981) studied a spherical aquifer-well system and obtained solutions similar to that of Bodvarsson. Robinson and Bell [1971] applied Bredehoeft's technique to the analysis of earth tide response in six wells from

the Appalachian region. Marine [1975] applied Bredehoeft's approach to the analysis of data from wells near Aiken, South Carolina, and found practical difficulties in estimating the formation parameters. Ardity *et al.* [1978] applied a modified form of Bodvarsson's [1970] solution to the interpretation of data from five oil and gas reservoirs.

The present study relates to the analysis of earth tide data from three geothermal reservoirs in the United States. In the analysis the aquifers are conceptualized in much the same way as in the work by Bredehoeft [1967], as an areally infinite, homogeneous, confined medium. However, on reexamining Bredehoeft's formulation, a key assumption is found to be suspect. The problem is therefore formulated somewhat differently. Before the field data were interpreted for elastic parameters, they were first spectrally analyzed to separate out the fundamental harmonic components.

The analysis is primarily based on principles of static equilibrium. However, it is known that phase differences in the form of time lags can exist between the actual fluid pressure response within the aquifer and the corresponding signal in the well. Hypothetical, numerical calculations were carried out to investigate the nature of these time lags.

This paper may be broadly divided into three parts. In the first the problem is defined, and the solution procedure is formulated. In the second the spectral analysis methodology is described, and in the last the analyses of the actual field data and the phase difference effects are discussed.

PROBLEM FORMULATION

The Static Problem

Consider a homogeneous, horizontal, areally infinite, confined aquifer penetrated fully by a well of radius r_w (Figure 1). Let the well be under shut-in artesian pressure conditions or under packed-off conditions so that the barometric pressure changes do not directly communicate with the fluid in the well. Or, for a well with a free water surface, assume that there is no change in barometric pressure. The well-aquifer system is under hydrostatic conditions. Further assume that the well is

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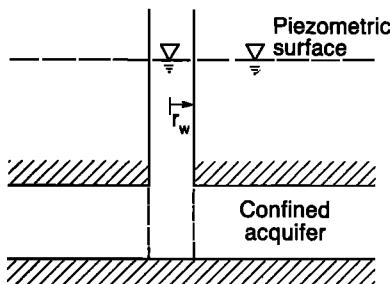


Fig. 1. Schematic representation of an areally infinite confined aquifer.

hydraulically perfect and instantaneously reacts to and equilibrates with any fluid pressure change in the aquifer.

The aquifer is now subjected to periodic changes in its in situ tectonic stress due to the transit of the sun and the moon. The stresses are accompanied by dilation of the aquifer. The fluid pressure in the aquifer rises and falls in response, the fluid pressure fluctuation being governed by the relative magnitudes of compressibilities of the porous medium and the water. The areal dimensions of the aquifer are small in comparison to the size of the portions of the earth equally affected by the earth tides. Hence, over large areas around the well, the magnitude of the stress changes (and hence the fluid pressure changes) are everywhere the same. Thus the gradient of fluid potential induced is negligible, and hence the lateral movement of water in the aquifer may be neglected. Even near the well, such movement, if any, may be neglected since the movement is very small and the well is assumed to sense instantaneously the fluid pressure changes in the aquifer. As a consequence of the lack of lateral water movement, the aquifer behaves as a whole as an undrained system.

Let $\delta\sigma_m$ be the mean principal stress change associated with the earth tides. Let δp be the pore pressure change generated in response. The magnitude of δp relative to $\delta\sigma_m$ depends on the relative compressibilities of the porous matrix and water since undrained conditions demand that the pore volume change be identical to change in water volume. As is common in hydrogeology, we will assume that the rock grains are essentially incompressible as compared to the compressibility of the porous medium. Then, subject to the condition

$$\delta\sigma_m = \delta\sigma_m' + \delta p \quad (1)$$

where $\delta\sigma_m'$ is the mean principal effective stress borne by the porous matrix, the dilatation of the matrix and that of water shall be equal. Thus

$$\Delta = -c_m \delta\sigma_m' \quad (2)$$

where Δ is the dilatation of the porous matrix, positive in expansion and negative in contraction, and c_m is the bulk compressibility defined as $c_m = -d\phi/d\sigma_m'$, where ϕ is porosity. Also, the dilatation of water, Δ_w , equals

$$\Delta_w = -\phi c_w \delta p \quad (3)$$

where c_w is the compressibility of water. Under undrained conditions, $\Delta = \Delta_w$ and in view of (1), (2), and (3),

$$\frac{\delta p}{\delta\sigma_m} = \frac{1}{1 + (\phi c_w / c_m)} \quad (4)$$

The ratio $\delta p/\delta\sigma_m$ is sometimes referred to as the tidal efficiency. In the earth tide problem, δp is measured and c_w is

known with reasonable certainty. There remain three unknowns, $\delta\sigma_m$, ϕ , and c_m , of which the last two are aquifer parameters. Indeed, if ϕ and c_m are known, S_s can be easily computed by the well-known equivalence

$$S_s = \gamma_w(\phi c_w + c_m) \quad (5)$$

where γ_w is the unit weight of water and S_s is the coefficient of specific storage. Or the pore volume compressibility c_p and the "total compressibility" c_t could be estimated by the relations

$$c_p = c_m/\phi \quad (6)$$

$$c_t = c_w + c_p \quad (7)$$

It may be noted here that S_s and c_t are related to each other by

$$c_t = S_s/\gamma_w\phi \quad (8)$$

$$S_s = \gamma_w\phi c_t \quad (9)$$

Now going back to (4), the earth tide response provides directly the quantity δp . Data on $\delta\sigma_m$ are essential if we need to calculate c_p , c_t , or S_s . But $\delta\sigma_m$ is not directly forthcoming from earth tide data, and hence (4) is a priori apparently of limited use. The problem of indirectly estimating $\delta\sigma_m$ from earth tide data will be considered later.

For the moment, it is assumed that (4) is of limited use and that the problem must be looked at in other ways. In this connection, suppose that a two-stage experiment is considered in which a fully saturated porous material of volume V_0 is initially under hydrostatic conditions with initial pore pressure p_0 . Further suppose that this volume element is subjected to a stepwise external load. The two stages of the experiment are as follows:

1. Impose the stepwise load. Allow no drainage. Monitor the change in pore pressure δp .

2. Allow drainage so that the excess pressure δp dissipates and the pore pressure over V drops back to its original value p_0 . Measure the new volume V after drainage or measure the volume ΔV_w of the amount of water drained.

It will now be shown that the information collected in stages 1 and 2 can be used together to determine S_s . In stage 1 the dilatation undergone by the matrix $(-\phi c_w \delta p)$ is equal to the dilatation undergone by the fluid which is $-\phi c_w \delta p$. During the second stage the decay of δp imposes an additional effective stress change in the matrix equal in magnitude to δp with the matrix undergoing a further dilatation equal to $-c_m \delta p$. The total dilatation that the matrix has undergone during both stages equals $-\{c_m(\delta\sigma_m - \delta p) + c_m \delta p\} = -c_m \delta\sigma_m$. This dilatation, incidentally, is also equal to the volume of water drained during stage 2 per unit bulk volume of the element. Thus

$$\Delta_{drained} = \frac{V - V_0}{V_0} = \frac{\delta V_w}{V_0} = -(\phi c_w + c_m) \delta p = -c_m \delta\sigma_m \quad (10)$$

According to (10), measuring δp and $\Delta_{drained}$, where $\Delta_{drained}$ is the total "dilatation" subsequent to drainage, allows for the computation of S_s by the relation

$$S_s = -\gamma_w \Delta_{drained}/\delta p = \gamma_w(\phi c_w + c_m) \quad (11a)$$

Or, recognizing that $\delta p = \gamma_w \delta h$, we may write

$$-\delta h = \Delta_{drained}/S_s \quad (11b)$$

Equation (11b) is very similar to the final expression arrived at by Bredehoeft [1967, equation (25)] if we consider that $\Delta_{\text{drained}} \equiv \Delta_t$. However, some difficulties arise.

In his development, Bredehoeft assumes that the tidal strains that define Δ_t are independent of the elastic properties of the aquifer and are almost entirely determined by the elastic properties of the earth as a whole [Bredehoeft, 1967, p. 3081]. Yet, in his equation (25) (equation (11b) above), Δ_t is related to S_s , which is clearly a function of the aquifer parameters. There is a contradiction here. Whereas Bredehoeft assumes that Δ_t is independent of the aquifer parameters, he does assume [Bredehoeft, 1967, equation (3), p. 3079] that Δ , the dilatation of a confined aquifer due to earth tides, is dependent on the aquifer parameters. The following question now arises: If Δ_t is independent of aquifer parameters, what causes the pore fluid pressure to change? The answer to this question is that contrary to Bredehoeft's assumption, the dilatation which an aquifer undergoes due to the stress changes accompanying the gravitational attractions of the sun and the moon is indeed a function of the aquifer parameters. The actual dilatation undergone by the confined aquifer due to earth tide response is

$$\Delta = \Delta_t + \Delta_h \quad (12a)$$

In (12a) the following definitions hold:

$$\Delta = -(\delta\sigma_m - \delta p)/E_s = -\phi\sigma p/E_w \quad (12b)$$

$$\Delta_h = \delta p/E_s \quad (12c)$$

$$\Delta_t = -\delta\sigma_m/E_s \quad (12d)$$

In the above we use the convention that $\Delta > 0$ implies expansion and $\Delta < 0$ implies contraction. Note in particular that (12c) differs from equation (22) of Bredehoeft in that the signs are reversed. Indeed, Bredehoeft's equation (22) is not consistent with his equations (3), (21), and (23). For, according to his equation (21) we may write

$$\Delta_t = \Delta - \Delta_h \quad (13)$$

Now if we substitute for Δ from his equation (3) and for Δ_h from his equation (22), we get

$$\Delta_t = -\frac{n\rho dh}{E_w} + \frac{n\rho dh}{E_s} \quad (14)$$

which differs from Bredehoeft's equation (23) in that the sign attached to the $n\rho dh/E_s$ term is reversed.

Finally, the basic implication in Bredehoeft's development is that one can simultaneously measure (1) "... the dilatation that would occur if the fluid were not present" [Bredehoeft, 1967, p. 3080] and (2) the pore pressure generated due to the earth tides. That is not possible because the two events are mutually exclusive. Rather, we interpret the statement, "... the dilatation that would occur if the fluid were not present" to mean, "if the pore pressure were to remain unchanged." In a fluid-saturated confined porous medium if we were to maintain the condition that the dilatation of the water is equal to that of the porous medium [Bredehoeft, 1967, p. 3078], then the pore fluid pressure cannot remain unchanged. On the contrary, if we were to keep the pore fluid pressure unchanged and still have dilatation of the matrix, then we cannot satisfy the condition that the dilatation of the water is equal to that of the porous medium. Indeed, if one were to measure the dilatation of the earth using a suitable strain gauge, then the contribution of the aquifer to that global dilation shall be the undrained dilatation of the aquifer and not the "dilatation

that would occur if the fluid were not present," as assumed by Bredehoeft.

For this reason, the approach of Bredehoeft is not followed in the present work. Instead, an alternative assumption is devised to estimate c_p and c_t rather than S_s , as described below.

In view of the above discussions it is clear that the measurement of δp alone is inadequate to interpret reasonably the earth tide response. The problem becomes tractable if in addition to δp the in situ tectonic stress changes can also be measured. It would provide a direct measure of $\delta\sigma_m$ in (4). Although such sensitive stress measurements are currently not carried out, they may become possible in the future with the availability of improved instrumentation.

Estimation of c_p and c_t .

In view of the fact that the observed response of the aquifer to earth tides is an undrained one, S_s , which is fundamentally a parameter defined only for the drained problem, cannot be directly computed. This leaves (4) to be examined once again. In looking at (4), a means of estimating $\delta\sigma_m$ must be considered. Since tidal forces induce stresses in the earth, it is reasonable to assume that equilibrium tide theory can be used to compute mean stress changes caused by earth tides rather than dilatation. It would be ideal if this quantity can be reasonably estimated for the region in the vicinity of the aquifer. With a view to pursuing this conceptual approach, the following alternate reasoning is employed. It is assumed that Δ_t , calculated according to Takeuchi [1950] ($\Delta_t = 0.5 (W_2 ag)$, where W_2 is tidal potential, a is the radius of earth, and g is acceleration due to gravity), can be used in conjunction with an appropriate estimate of compressibility or bulk modulus to yield $\delta\sigma_m$. Thus

$$\delta\sigma_m = K\Delta_t \quad (15)$$

where K is an estimated value of bulk modulus. The question now arises as to what is an appropriate value of bulk modulus. Should it reflect the rocks making up the aquifer or should it be taken as the value obtained from the earth model that the tidal potential is applied to? The answer should apparently be somewhere in between. It might be expected that aquifer materials would generally have lower bulk moduli than other rocks at a given depth and hence the bulk modulus in and around the aquifer may be lower than what might be expected from a global point of view. However, considering the heterogeneities of geologic systems, there is no physical way by which the appropriate bulk modulus in and around the aquifer may be precisely defined. Yet, as is common with interpreting other field data, reasonable assumptions can be made to arrive at satisfactory inferences. As evidenced by Bredehoeft [1967], the best estimate for the bulk modulus from the earth model of Takeuchi [1950] is 1×10^{11} Pa. This rather high value approximates that of the ultrabasic rock, dunite. Since aquifers are closer to the earth's surface and more porous, they are likely to have smaller bulk moduli. Hence, 1×10^{11} Pa could be taken as an upper limit in our case. Values of bulk moduli for some common rocks are given in Table 1. As can be seen, most of the values are substantially below 1×10^{11} Pa. Since the deformation being considered is on a large scale, the relatively low bulk modulus of the aquifer should be expected to affect but not totally govern the system. The best value of bulk modulus should therefore lie somewhere between the extremes. Accordingly, a value of 5×10^{10} Pa is chosen to be a reasonable estimate for K in the general vicinity of deep, confined aquifers. In view of this, (15) may be

TABLE 1. Representative Bulk Moduli for Some Common Rock Types

Rock Type	Rock Modulus, Pa
Basalt*	7.5×10^{10}
Dunite*	1×10^{11}
Granite*	5.5×10^{10}
Sandstone	1.7×10^{10}
Iron (solid)	1.6×10^{11}

*From Stacey [1969].

substituted into (4) to obtain

$$c_p = \frac{\delta p c_w}{K_t - \delta p} \quad (16)$$

$$c_t = c_w + c_p = S_s / \gamma_w \phi \quad (17)$$

where $K = 5 \times 10^{10}$ Pa. In the interpretation of actual field data to be presented later, we shall use (16) and (17) instead of the procedure adopted by Bredehoeft [1967]. Even this approach is not completely satisfying. The overall problem can best be solved only if $\delta\sigma_m$ can be measured in situ in the field in addition to δp .

Earth Tides and Barometric Fluctuations

One of the suggestions made by Bredehoeft [1967] is that once S_s is estimated from earth tide effects, it is then possible to use the S_s estimate in conjunction with barometric efficiency to estimate porosity. This implies that the earth tide response contains information not contained in the barometric response and vice versa. The problems associated with this suggestion are examined below.

Consider first the earth tide problem. Here, the confined aquifer is subject to an external stress $\delta\sigma_m$ under undrained conditions. The external stress, however, does not act on the water level in the well. The excess fluid pressure δp generated within the aquifer creates a difference in potential between the aquifer and the well and water flows into the well. If $\delta\sigma_m$ is positive (compression), δp is positive and the water level rises in the well and vice versa. Also, since δp represents the portion of $\delta\sigma_m$ borne by the water, the rise in fluid pressure or water level in the well is an indication of the stress borne by the water.

Now consider the barometric problem. Again the confined aquifer is subject to an external stress. This stress is a vertical stress $\delta\sigma_v$ (rather than $\delta\sigma_m$). If one can assume that barometric stresses are relatively uniform over large areas, then $\delta\sigma_v$ will be felt almost completely at the top of even deep, confined aquifers. Insofar as the generation of pore pressure in the aquifer is concerned, the aquifer will respond in this case too in an undrained fashion, resulting in a pore pressure generation δp just as in the earth tide case. Unlike the earth tide case, however, the external stress $\delta\sigma_v$ also acts directly on the fluid level in the well. Since $\delta\sigma_v > \delta p$, an imbalance is created in the fluid pressure at any elevation in the well-aquifer system. To overcome this imbalance and reestablish hydrostatic equilibrium, the water level will have to decline in the well (if fluid pressure had increased in the aquifer) and vice versa by an amount equal to $(\delta\sigma_v - \delta p)/\gamma_w$. Note that $(\delta\sigma_v - \delta p)$ is the portion of the external stress borne by the porous matrix under undrained conditions. Thus the change in water level is directly a measure of the stress borne by the porous matrix under undrained conditions. If, however, the well is packed-off

or sealed so that the fluid level has no communication with the atmosphere, then the mechanisms of response in the well will be identical in the earth tide and barometric cases. Except for the differences in the manner in which external stresses acts on the water level in the well and the minor difference in the nature of the external load ($\delta\sigma_m$ versus $\delta\sigma_v$), the mechanisms governing the earth tide and the barometric problems are identical. Bredehoeft's effort to estimate S_s from the earth tide case and to use it in the barometric response to determine porosity is a consequence of his assumption that Δ_c , computed theoretically from the equilibrium tide methods, represents the drained response of the aquifer. If this assumption is invalid, then the use of earth tide or barometric data will lead to the estimation of the same set of aquifer parameters.

As a matter of detail, there is one notable difference between earth tide and barometric responses. In the barometric case the loading is effectively one dimensional since $\delta\sigma_v$ acts over a large area. Hence horizontal strains can be neglected, and deformation is primarily purely vertical. The resulting compressibility is of the "odometer-type," represented by the coefficient of volume change [Lambe and Whitman, 1969], $m_v = -\epsilon_v/d\sigma'$, where ϵ_v is the vertical strain. On the contrary, the earth tide case is characterized by a general three-dimensional stress field in which an assumption of one-dimensional deformation will not be strictly valid. A generalized bulk compressibility defined with reference to mean principal stress change (that is, $c_m = -d\phi/d\sigma'$) is used. In principle, m_v and c_m will not be the same for a given material. At the same time, it is also not clear as to which of these terms should enter into the definition of specific storage. Although an attempt to resolve this question is not made in this paper, it should be recognized that it will be worthwhile in the future to investigate whether combined earth tide/barometric analysis could help evaluate the two different coefficients of compressibility.

Actually, it appears that the barometric response problem is far better posed than the earth tide problem. For, in the barometric problem, $\delta\sigma_v$ is known as well as δp : if the well has a free water surface, $\delta h = (\delta\sigma_v - \delta p)/\gamma_w$; or if the well is packed-off, δp equals the change in fluid pressure in the well. Therefore, whenever possible, it may be far more profitable to compute c_p and c_t according to the relations

$$c_p = \frac{(\delta p_A - \gamma_w \delta h)c_w}{\gamma_w \delta h} \quad (18)$$

$$c_t = c_w + c_p \quad (19)$$

Where $\delta p_A = \delta\sigma_v$ is the change in barometric pressure and δh is the change in water level in the well. For a packed-off well subject to barometric effects, (18) may be rewritten as

$$c_p = \frac{\delta p c_w}{\delta p_A - \delta p} \quad (20)$$

The best method of analysis of any data containing earth tide and barometric effect is to run a power spectrum analysis of the fluid pressure data to separate the earth tide components and the barometric variations. The spectrum analysis facilitates the comparison of the relative strength of the tidal related signal to barometric variations and random noise. It allows for identification of the dominant tidal components to be used in determining c_p and c_t . In situations where both earth tide and barometric influences are very discernible, it may be desirable to isolate the well from the atmosphere so that the complications arising out of the effect of one event on

the water level but not the other can be avoided. In fact, the response of the aquifer to extreme events during storms may provide valuable insights into the aquifer behavior. Once c_s and c_p are computed from the well response, one can use the computed values to evaluate the $\delta\sigma_m$ values for the earth tide problem. It is possible that these estimated $\delta\sigma_m$ values may help in a better understanding of the earth tide mechanisms.

In dealing with the tidal efficiency (TE) and barometric efficiencies (BE) it is always theoretically assumed that $TE + BE = 1.0$. It should indeed be of practical interest to observe the barometric response of the same well under free surface and packed off conditions to verify this. It should not be surprising if deviations are found, since BE is governed by one-dimensional compressibility and TE is governed by three-dimensional compressibility.

The Dynamic Problem: The Role of Well Bore Storage

In the foregoing discussions we have assumed that the well is hydraulically perfect and that it instantaneously equilibrates to any fluid pressure change in the aquifer. This assumption will be of practical meaning only when the aquifer has high transmissivities and the specific storage of the well S_{sw} , defined as the volume of water released from the well per unit change in pressure head, is small. For a well with fluctuating water level $S_{sw} = \pi r_w^2$. When transmissivities are low and r_w is large, there will be a time lag between the event in the aquifer and the response in the well. Where the pore pressure event in the aquifer is of a periodic nature, the time lag will be manifested as a phase difference between the signals. In addition, the signal in the well will also undergo damping. Phase lags and damping could also occur because of the inertial effects of the moving water column in the well. Bredehoeft [1967] concluded that for earth tides and barometric fluctuations the damping caused by inertial effects could be reasonably neglected. The effect of well bore storage on phase lag and damping on the other hand are not necessarily small.

Well bore storage effects on the earth tide response of wells has been documented. Marine [1975] reports that in a deep boring in a low permeability formation, the earth tide response was considerably increased when the well was packed off. Melchior et al. [1964] analyzed data from a well near Basecles, Belgium, and found a phase shift of up to 25° between the theoretical dilatation produced by individual waves and the harmonic components reduced from field data. They found it difficult to explain this phase shift.

That phase lags and damping effects could be observed is encouraging. The possibility exists that one could use these additional data to analyze the tidal response in a more detailed fashion. Nevertheless, analysis of natural periodic phenomena to obtain phase lag information can often pose practical difficulties. For example, Dale [1974] did a detailed study of daily tidal response from the ocean as well as the corresponding water level changes in a well. He analyzed different segments of the data over a year and found that the standard deviation of tidal efficiency was about 10% of the mean but that the standard deviation of phase lag was over 200% of the mean. It is possible that better filtering techniques may help improve the situation. It is, however, still of interest to investigate how the phase lag and damping information may be analyzed to obtain aquifer parameters.

The problem may be conceptualized as a well penetrating an infinite homogeneous aquifer. Everywhere within the aqui-

fer, pore pressure is being generated at the same rate except at the well. A transient pressure regime is therefore set up in a small region around the well. This region is ill-defined, and its radius will depend on the aquifer transmissivity T and storage S . A simple way to solve this problem analytically is to assume that the radius r_0 of the transient region is known and evaluate the response function $f(t)$ at the well if the pressure is changed by unity at r_0 . Then, if $g(\tau)$ denotes the periodic source generation function, the fluctuation of water level $F(t)$ in the well may be expressed as the convolution,

$$F(t) = \int_0^t g(\tau) f(t - \tau) d\tau \quad (21)$$

A second and more general way to evaluate the solution analytically would be to solve the transient diffusion equation in the aquifer with a periodic source term that is spatially constant within the aquifer but is zero in the well. This condition is, in a sense, complementary to the conventional pumping test problem in which the source term is zero in the aquifer and nonzero in the well.

In the present study a numerical technique for analyzing the phase lag problem was chosen in preference to the analytical approach because the numerical approach provides greater flexibility and generality in handling the complex conditions of the problem.

Parametric Studies on the Dynamic Problem

Inasmuch as reliable phase lag field data were not available, a parametric study on a hypothetical problem was carried out to gain insight into the mechanisms controlling phase lag and damping in the well. The numerical model used in this work is an integral finite difference program, TERZAGI. It is a multi-dimensional model for saturated flow involving time-dependent parameters. This model is a simplified version of a more general model presented elsewhere [Narasimhan et al., 1978; Narasimhan and Witherspoon, 1978]. The special features that the model had to consider for the problem under consideration included a well with prescribed storage capacity and a prescribed periodic generation of pore pressure everywhere within the flow region. The model has been extensively validated [Narasimhan and Witherspoon, 1978] and was further validated for this problem by comparing it to the analytic solution for the slug test problem [Cooper et al., 1967]. The effects of the following parameters: well bore storage, period of the source term, permeability, and specific storage. In all the calculations a simple sine wave was used to represent the periodic source term.

Effect of well bore storage. An aquifer of thickness 10 m, absolute permeability 10^{-14} m^2 , and $S_s = 10^{-4} \text{ m}^{-1}$ was considered. It is penetrated by a well of radius 0.2 m which is completely shut-in. The storage coefficient for this well is $\pi r_w^2 c_w H \gamma_w = 6 \times 10^{-6} \text{ m}^{-1}$, which is indeed very small. The response of this well to the sinusoidal source term is shown in Figure 2a. As can be seen, the heads in the well are extremely close to those in the aquifer, indicating almost instantaneous well response. In Figure 2b shows the case when the well is open to the atmosphere, which increases S_{sw} from 6×10^{-6} to $1.26 \times 10^{-1} \text{ m}^{-1}$. It is clear that the well response shows some phase lag as well as damping. In both Figures 2a and 2b the solid line represents the signal at a point 90 m away from the well.

Effect of source term periodicity. Since phase lag is governed by a convolution integral, it would be expected that

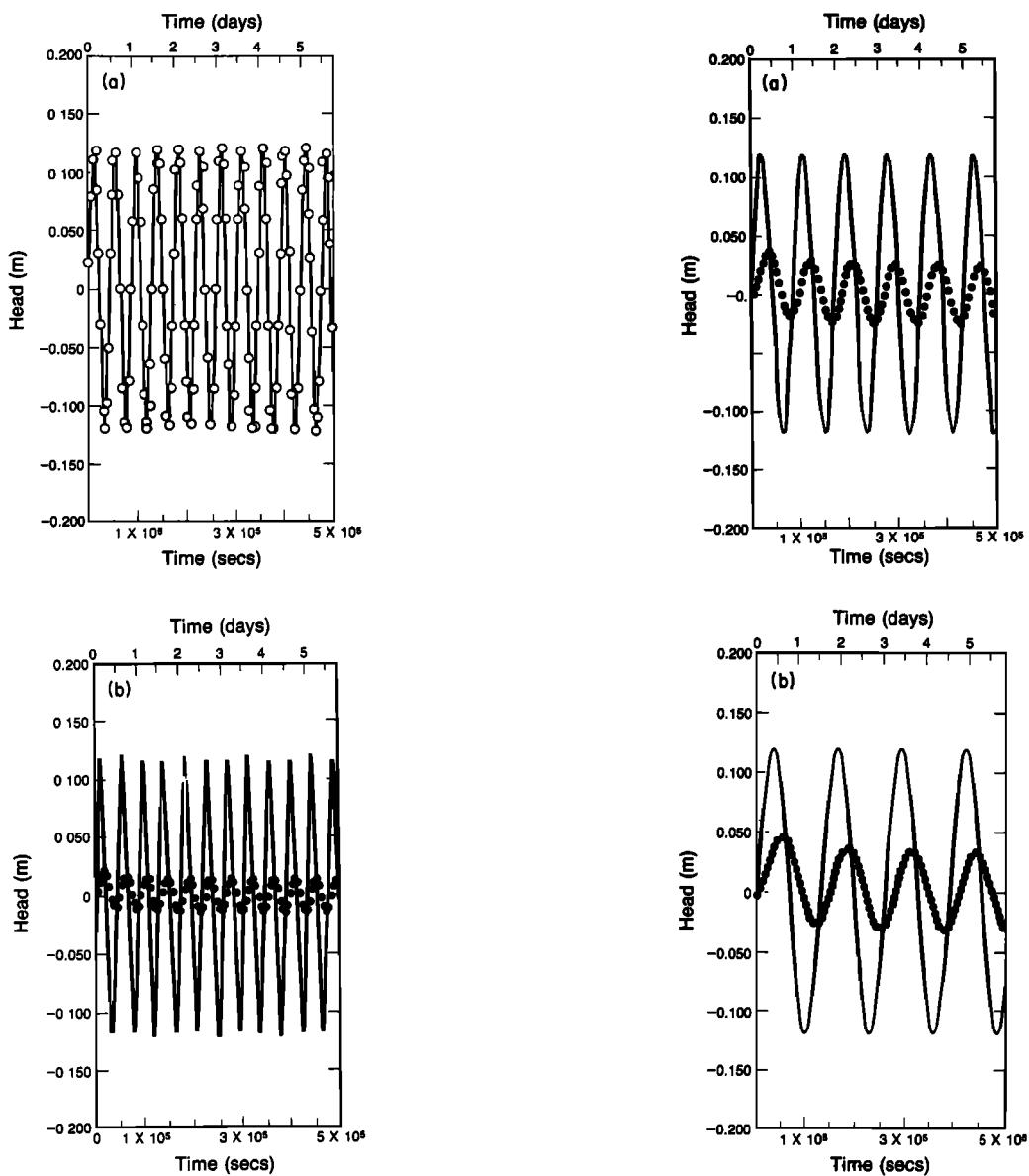


Fig. 2. Effect of well bore storage on fluid pressure in the well: (a) shut-in well, well bore storage dictated by compressibility of water in the well bore, and (b) well with free water surface, well bore storage dictated by radius of the well bore.

both the phase lag and the damping will be related to the periodicity of the source term. This relationship was checked by changing the periodicity of the source term from 12 to 48 hours. The results are shown in Figures 3a, 3b, and 3c. As may be seen, the well response tends to agree more and more with the source term as the period increases. The longer the period the smaller are the phase-lag and damping. The practical implication is that the response of the well better reflect the longer period tidal effects than the shorter period events.

Effect of permeability and specific storage. In order to evaluate the effect of permeability, a signal with an 18-hour (64,800 s) period was used, and the well was assumed to be open to the atmosphere. The permeability was varied from 10^{-14} to 10^{-13} m^2 with specific storage held constant at $1 \times 10^{-4} \text{ m}^{-1}$. The results are compared in Figures 4a, 4b, and 4c. The comparison shows, as expected, that increasing permeability reduces phase lag and damping.

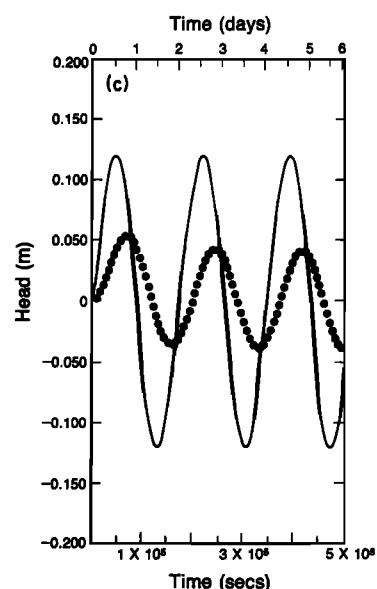


Fig. 3. Effect of source term periodicity on pressure response in the well: (a) 24 hours, (b) 36 hours, and (c) 48 hours.

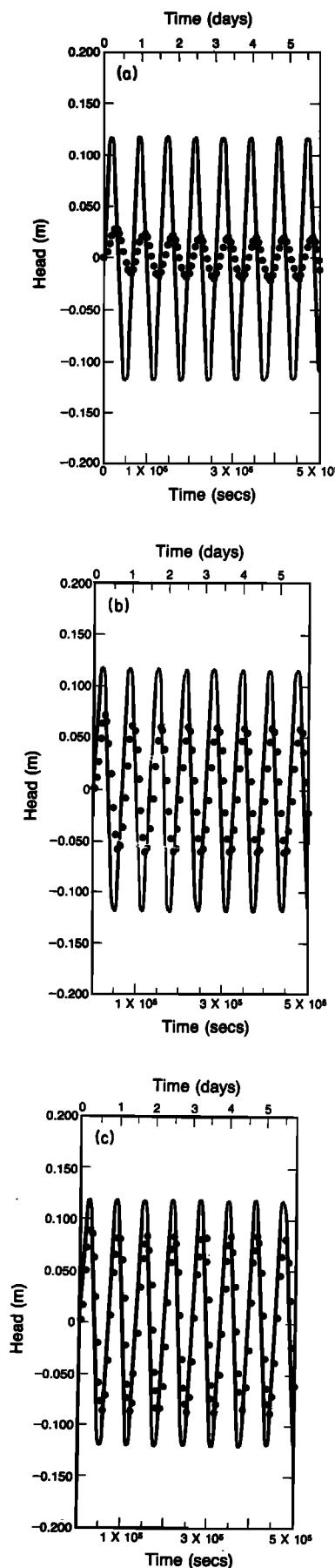


Fig. 4. Effect of permeability on pressure response in the well: (a) 10^{-14} m^2 , (b) $5 \times 10^{-14} \text{ m}^2$, and (c) 10^{-13} m^2 .

In another series of experiments, permeability was held constant at 10^{-14} m^2 , while specific storage was varied from 10^{-4} to 10^{-6} m^{-1} for the 18-hour period case. The results of these are summarized in Figures 5a, 5b, and 5c. Figure 5 indicates that decreasing S_s tends to increase phase lag and damping. However, the increase in damping and phase lag is relatively small, as S_s is decreased by two orders of magnitude, suggesting that they are relatively insensitive to changes in S_s .

In summary, the parametric studies show that decreasing well bore storage or increasing permeability, specific storage, and wave length will decrease phase lag and damping. The basic model of interpretation, however, involves a static response of an undrained system to external load. The best method of experimentation, therefore, may involve two steps. First the well is packed off and the compressibilities c_p and c_t are evaluated. Then the well is opened to the atmosphere and the phase lag and damping are measured. Now, if porosity can be estimated independently, S_s can be computed from c_p , and using this, the formation transmissivity may be estimated by an iterative process.

The Possibility of a Type Curve Method for the Dynamic Problem

From the foregoing, two possibilities suggest themselves for analyzing the dynamic problem. In the first, one simply plots the ratio $\delta p_{\text{well}}/\delta \sigma_m$ or the tidal efficiency, estimated for each of the several tidal components whose periods are known. If the parametric results are correct, these points should asymptotically approach some limiting value as the period of the wave increases. Thus it would be ideal to look for earth tide effects of fortnightly or monthly durations for this purpose. The asymptotic values could then be used in the static analysis along with equation (4).

The second possibility is that of a type curve method. Suppose the static analysis yields some estimate of S_s . Then since r_w is known, one could compute, numerically, the phase lags and damping coefficients for various combinations of T and wave length and prepare a set of type curves for the variation of the damping coefficient $D = p/p_0$ versus $\log (TE_L)/r_w^2 S$ where E_L is the lag coefficient. The values of $\delta p/\delta \sigma_m$ computed from field data for various components of the earth tide are thus matched against a plot of $\delta p/\delta \sigma$ versus the observed time lag. Using the matched position T could be estimated.

ANALYSIS OF FIELD DATA

In this section, interpretation procedure developed for the static problem is applied to field data from three geothermal wells: from Marysville, Montana; East Mesa, California; and Raft River, Idaho. In all the cases the field data on water level/fluid pressure in the well and the theoretical earth tide computed for the particular sites were first spectrally analyzed for identification of the six dominant frequencies. These dominant frequencies and their periods are given in Table 2. The program used in the present study for spectral analysis has been described by Kanehiro [1979]. Kanehiro [1977] also describes a program based on a filtering scheme developed by Lecolazet. The advantages of the fast Fourier transform over the filtering scheme is that the former shows the energy contained in the nontidal as well as the tidal components, whereas filtering schemes tend to concentrate only on the tidal components. The theoretical earth tide fluctuations for the particular sites over the particular durations of the observations were computed using a computer code developed by Harrison

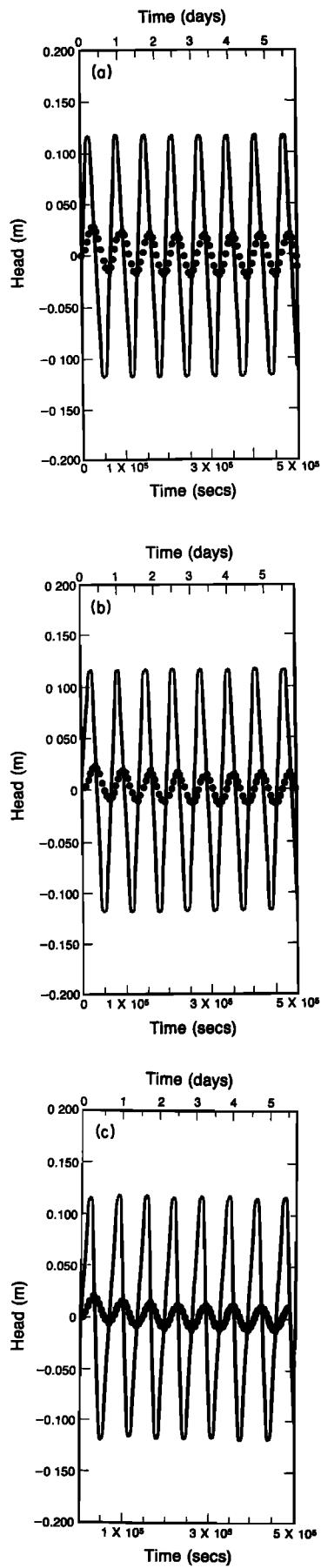


Fig. 5. Effect of specific storage S_s on pressure response in the well: (a) 10^{-4} m^{-1} , (b) 10^{-5} m^{-1} , and (c) 10^{-6} m^{-1} .

TABLE 2. Dominant Frequencies and Their Periods

Symbol	Name	Period, hours
K_1	lunisolar	23.934469
O_1	larger lunar	25.819341
P_1	larger solar	24.065891
M_2	principal lunar	12.420601
N_2	longer lunar elliptic	12.658348
S_2	principal solar	12.000000

[1971]. Experience indicates that for a reliable analysis it is desirable to have at least 28 days of continuous data.

The three wells considered below penetrate fractured, liquid-dominated reservoirs. For purposes of interpretation these reservoirs have been treated grossly as porous media, constituting confined aquifers. In addition, the possible effects of temperature on the observed pressure changes have been neglected.

Well at Marysville, Montana

This well, for which data were kindly provided by R. B. Leonard of the U.S. Geological Survey, is located about 30 km NW of Helena, Montana. It is 2000 m deep with an uncased portion 0.2 m in diameter in a fractured quartz porphyry. The well had a free water level, which was monitored with a float-type recorder. As has already been suggested in discussing the dynamic problem, there is reason to believe that the hydrograph data may be influenced by well bore storage. Nonetheless, we shall carry out the interpretation as if no well bore storage effect existed. The hydrograph record from the well as well as the corresponding earth tides are presented in Figure 6. Even visually one can recognize the correlation that exists between the two patterns. The power spectra of the two records are shown compared in Figure 7. The angular velocity ω or the period T of the tidal component is related to the harmonic number k by

$$\omega = 2\pi k/L \quad (22)$$

$$T = L/k \quad (23)$$

where L is the length of the record considered. It can be seen from Figure 7 that the spectra show large amplitudes for the same components. Since the wells are far from the ocean, all the fluctuations were attributed to earth tides. Assuming a fluid density of 1000 kg/m^3 , $g = 9.8067 \text{ m/s}^2$, and $c_w = 4.6 \times 10^{-10} \text{ M}^2/\text{N}$, one could compute S_s/ϕ as follows. For each of the six components we first compute Δ_t by the relation

$$\Delta_t = 0.5W_2/ag \quad (24)$$

where a is the radius of the earth. Then, assuming an appropriate value of K , bulk modulus for the region around the aquifer S_s/ϕ is estimated by the relation

$$\frac{S_s}{\phi} = \rho_w g c_w \left(1 + \frac{\delta p}{K \Delta_t - \delta p} \right) \quad (25)$$

For an assumed value of $K = 5 \times 10^{10} \text{ Pa}$, the computed values S_s/ϕ are presented in Table 3. The mean value of S_s/ϕ obtained is $1.14 \times 10^{-5} \text{ m}^{-1}$. Although no independent check is available, the magnitude appears reasonable. However, there is reason to suspect that the data are affected by well bore storage effect. Evidence is shown in the fact that the 24-hour components of the tide O_1 , P_1 , and K_1 show a larger response of δp and hence higher S_s/ϕ than the 12-hour com-

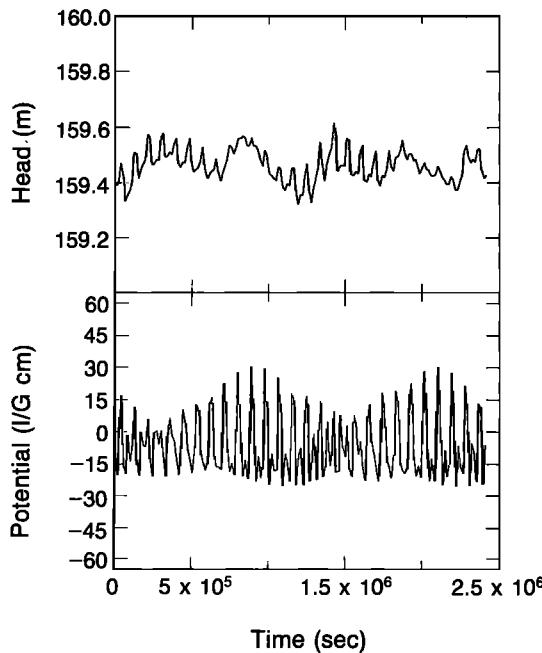


Fig. 6. Hydrograph of well at Marysville, Montana (top), and the corresponding earth tide variations (bottom).

ponents. This is due to the longer time available for the well to equilibrate in the case of the 24-hour components than the 12-hour components. In general, therefore, by neglecting well bore storage and using the static interpretation, one would tend to underestimate S_s/ϕ .

Well at East Mesa, California

The East Mesa geothermal well is located about 30 km east of El Centro, in the Imperial Valley of California. The valley is part of a large sediment-filled depression, the Salton Trough. The geothermal reservoir occurs within poorly consolidated

sandstones, siltstones, and shales of Tertiary age, derived from the Colorado River. The aquifer is confined on the top by a predominantly shaly sequence, about 600 m thick. The well is about 2000 m deep with a slotted interval of about 250 m. This shut-in well was monitored continuously at the well head by a continuous recording quartz crystal pressure transducer. The bottom-hole temperature was about 320°F. The field is approximately 100 km from the Salton Sea and 130 km from the Pacific Ocean. Although the tides in these large bodies of water might have an effect on the well response, it was assumed that such effects would be relatively small, and they were neglected.

The data from this well were analyzed by the same method as the previous example. The results are summarized in Table 4 for two different values of K . As can be seen, for an assumed bulk modulus of $K = 5 \times 10^{10}$ Pa, the mean $S_s/\phi = 5.2 \times 10^{-6}$ m $^{-1}$ and for $K = 10^{11}$ Pa, mean $S_s/\phi = 4.8 \times 10^{-6}$ m $^{-1}$. Independent pumping tests conducted over the East Mesa field [Narasimhan et al., 1977] suggest variations of the storage coefficient S from 2×10^{-4} to 1×10^{-3} , with a mean of about 6.2×10^{-4} . In order to compare our estimates with S we must have an estimate of the thickness-porosity product for the aquifer. Assuming a thickness of 1000 m for the aquifer, based on geologic considerations, and $S = 6.2 \times 10^{-4}$, the computed values of S_s/ϕ suggest a porosity of about 12%. This relatively reasonable estimate of porosity points to the validity of this analysis.

Well at Raft River, Idaho

The Raft River Valley well analyzed is about 24 km south of Malta, Idaho, within a valley 60 km long and 19 km wide opening to the north. The valley is graben filled with Tertiary sediments and volcanics to a depth of about 1800 m. The rocks are mainly made up of tufaceous sandstones, siltstones, and conglomerates.

The well under consideration is 1525 m deep with an estimated production zone of about 175 m. This estimate refers to the open interval in the well, and the actual thickness is probably larger but is very difficult to estimate. The bottom-hole temperature in this shut-in well was about 295°F. Fluid pressure data from this shut-in artesian well were collected with a quartz crystal pressure transducer. The data, however, presented difficulties for spectral analysis for two reasons. First, there were many gaps in the record because of instrumentation failure. Second, the data were collected during a long-term interference test. Hence the data contained a long-term effect. For these reasons, attempts at spectral analysis were not very fruitful. A cruder method of peak-to-peak estimation was therefore employed. The double amplitude of a given peak was measured in the well record and compared with the double amplitude of the corresponding tidal potential. This can be done for any number of peaks and the mean value of the final results of the calculation used. Only the clean portion of the continuous well record was used. The results of the analysis are summarized in Table 5. As can be seen, the mean value of S_s/ϕ is 1.13×10^{-5} m $^{-1}$ for lower K and 6.43×10^{-6} m $^{-1}$ for higher K . The standard deviations are also small.

Independent, long-duration interference tests at Raft River [Narasimhan and Witherspoon, 1977] indicate $\phi c H$ values (where H is aquifer thickness) in the range of 4.15×10^{-8} to 5.26×10^{-8} m/Pa, while short-term tests indicated a higher range, varying from 3.18×10^{-7} to 1.13×10^{-6} m/Pa. The long-term interference test result are considered to be more

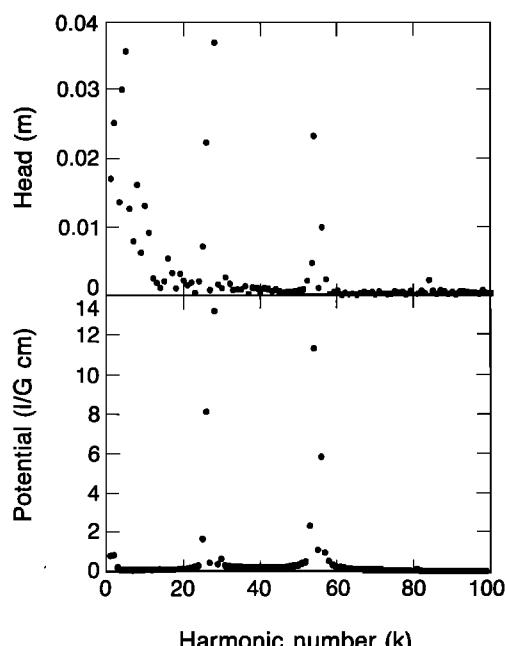


Fig. 7. Spectrum of amplitudes for the Marysville, Montana, well hydrograph (top) and the corresponding earth tides (bottom).

TABLE 3. Well at Marysville, Montana, Summary of Analysis

Tidal Component	$W_2/g, \text{ m}^{-1}$	Δ_t	$\delta p, \text{ Pa}$	Tidal Efficiency $\delta p/K\Delta_t$	$S_s/\phi, \text{ m}^{-1}$
O_1	8.126×10^{-2}	1.276×10^{-8}	2.176×10^2	0.34	1.485×10^{-5}
P_1, K_1	1.318×10^{-2}	2.07×10^{-9}	3.581×10	0.35	1.535×10^{-5}
N_2	2.315×10^{-2}	3.635×10^{-9}	4.6×10	0.25	9.351×10^{-6}
M_2	1.133×10^{-1}	1.780×10^{-8}	2.26×10^2	0.25	9.366×10^{-6}
S_2	5.863×10^{-2}	9.208×10^{-9}	9.782×10	0.21	7.962×10^{-6}

Assumption: $K = 5 \times 10^{10} \text{ Pa}$. S_s/ϕ (mean) = 1.14×10^{-5} . Standard deviation $3.45 \times 10^{-6} \text{ m}^{-1}$.

realistic on a large scale. They yield storage coefficient ($S_s \times H$) in the range of 4.07×10^{-4} to 5.16×10^{-4} . In order to compare the earth tide estimates of S_s with pump test, estimates of S_s , thickness H , and porosity ϕ are needed. If an average ϕ of 10% is measured, the mean S_s works out to $1.13 \times 10^{-6} \text{ m}^{-1}$, which leads to an effective aquifer thickness of about 440 m. This effective thickness agrees well with the geologic knowledge of the system.

DISCUSSION

From a phenomenological viewpoint there is little doubt that confined aquifers respond to earth tides. Assuming that the aquifer is elastic over the range of tidal stress variations and considering the dimensions of even large confined aquifers to be small compared to the areas of influence of the sun and moon, it is reasonable to conclude that the confined aquifer responds in an undrained fashion to the stress changes induced by earth tides. The physics of the undrained problem is such [Narasimhan and Kanehiro, 1980] that the dilatation of the aquifer is largely controlled by the compressibility of water. Therefore, if all the required data are forthcoming, namely, the change in total external stress, the pore pressure rise, the porosity, and the compressibility of water, then the undrained experiment can yield the compressibility c_m of the porous matrix. Using this, S_s can be computed by the relation $S_s = \gamma_w(\phi c_w + c_m)$.

It is important to recognize that by definition, specific storage S_s is a parameter defined only for the drained problem. Hence it is impossible to determine directly S_s from an undrained response of a confined aquifer to earth tides. Bredehoeft [1967] assumes that while the observed pore pressure

change due to earth tides is characteristic of an undrained condition, the tidal dilatation computed from the equilibrium tide theory of Love [1911] gives the dilatation of the aquifer after drainage and dissipation of the excess pore pressure. This assumption is questionable because a system, at a given instant, cannot behave both in an undrained and in a drained fashion. It appears that Δ_t estimated from equilibrium tide theory is a gross average for a large portion of an idealized earth, suitable for global geophysical calculations. It may not be appropriate for interpreting the response of confined aquifers.

A key parameter that is lacking in interpreting the undrained response of the confined aquifer is a direct determination of the magnitude of the externally imposed tidal stress $\delta\sigma_m$. Current procedures employing equilibrium earth tide theory provide estimates of dilatation Δ_t . Rather than attempting to evaluate $\delta\sigma_m$ directly, $\delta\sigma_m$ was estimated in the present study from Δ_t by assuming a reasonable value of bulk modulus for a relatively large region that includes the aquifer. Using this value, the pore volume compressibility c_p , the total compressibility c_m , and the quantity S_s/ϕ are estimated. It recognized that the approach employed, which involves estimating $\delta\sigma_m$, is not completely satisfactory. The earth tide response problem will not be well posed unless δ_p and $\delta\sigma_m$ are simultaneously measured within the aquifer.

A comparison of earth tide response with barometric response indicates that both phenomena provide the same amount of information regarding the elastic properties of the aquifer. From a practical viewpoint the barometric problem is better posed since the external stress $\delta\sigma_b$ is directly known, unlike the earth tide case. In the case of barometric analysis

TABLE 4. Well at East Mesa, California, Summary of Analysis

Tidal Component	$W_2/g, \text{ m}^{-1}$	Δ_t	$\delta p, \text{ Pa}$	Tidal Efficiency $\delta p/K\Delta_t$	$S_s/\phi, \text{ m}^{-1}$
$K = 5 \times 10^{10} \text{ Pa}$					
O_1	7.06×10^{-2}	1.109×10^{-8}	2.924×10	0.053	5.052×10^{-6}
P_1, K_1	7.362×10^{-2}	1.156×10^{-8}	4.745×10	0.082	5.416×10^{-6}
N_2	3.754×10^{-2}	5.896×10^{-9}	1.102×10	0.037	4.881×10^{-6}
M_2	1.704×10^{-1}	2.676×10^{-8}	5.526×10	0.041	4.923×10^{-6}
S_2	9.235×10^{-2}	1.450×10^{-8}	3.717×10	0.051	5.035×10^{-6}
$K = 1 \times 10^{11} \text{ Pa}$					
O_1	7.06×10^{-2}	1.109×10^{-8}	2.924×10	0.026	4.765×10^{-6}
P_1, K_1	7.362×10^{-2}	1.156×10^{-8}	4.745×10	0.041	4.920×10^{-6}
N_2	3.754×10^{-2}	5.896×10^{-9}	1.102×10	0.019	4.687×10^{-6}
M_2	1.704×10^{-1}	2.676×10^{-8}	5.526×10	0.021	4.706×10^{-6}
S_2	9.235×10^{-2}	1.450×10^{-8}	3.717×10	0.026	4.757×10^{-6}

S_s/ϕ (mean) = $4.77 \times 10^{-6} \text{ m}^{-1}$; standard deviation = $9.17 \times 10^{-8} \text{ m}^{-1}$.

TABLE 5. Well at Raft River Valley, Idaho

Peak Identification	W_2/g , m^{-1}	Δ_t	δp , Pa	Tidal Efficiency $P/K\Delta_t$	S_s/ϕ , m^{-1}
$K = 5 \times 10^{10} \text{ Pa}$					
1	3.9×10^{-1}	6.126×10^{-8}	9.655×10^2	0.32	1.265×10^{-5}
2	5.3×10^{-1}	8.326×10^{-8}	1.172×10^3	0.28	1.061×10^{-5}
3	6×10^{-1}	9.425×10^{-8}	1.172×10^3	0.25	9.165×10^{-6}
4	6.1×10^{-1}	9.582×10^{-8}	1.517×10^3	0.32	1.276×10^{-5}
5	5.6×10^{-1}	8.719×10^{-8}	1.310×10^3	0.30	1.151×10^{-5}
$K = 1 \times 10^{11} \text{ Pa}$					
1	3.9×10^{-1}	6.126×10^{-8}	9.655×10^2	0.16	6.648×10^{-6}
2	5.3×10^{-1}	8.326×10^{-8}	1.172×10^3	0.14	6.328×10^{-6}
3	6×10^{-1}	9.425×10^{-8}	1.172×10^3	0.13	6.043×10^{-6}
4	6.1×10^{-1}	9.582×10^{-8}	1.517×10^3	0.16	6.663×10^{-6}
5	5.6×10^{-1}	8.797×10^{-8}	1.310×10^3	0.15	6.479×10^{-6}

S_s/ϕ (mean) = $1.13 \times 10^{-5} \text{ m}^{-1}$; standard deviation = $1.5 \times 10^{-6} \text{ m}^{-1}$.

for an open well the fluid level as well as the aquifer is affected. In a shut-in well, however, both barometric changes and earth tides should provide the same kinds of response. Therefore the first step in this passive aquifer test procedure should be to measure fluid pressure in the packed-off well along with barometric pressures and earth tides. A spectral analysis of the fluid pressure fluctuations is then used to identify tidal component and barometric responses. Comparison of this response with the known tidal and barometric fluctuations allows for estimation of c_p , c_v , or S_s/ϕ . This experiment may later be followed by opening the well to the atmosphere. The time lags and damping associated with the open well signals can be used, under favorable conditions, to estimate aquifer transmissivity. The parametric studies show that finite well bore storage leads to a phase lag and attenuation of the aquifer signal with larger well bore capacity, lower permeability, and higher S_s leading to larger phase lag and greater attenuation. Some of the newer filtering schemes may help in better evaluating phase lag. Digressing a little, it would be interesting to analyze the barometric response in the same well under open and packed-off conditions and verify whether barometric and tidal efficiencies actually add up to 1, as postulated conceptually. The ideal time to make these measurements would be during periods of storm activity, when barometric pressure fluctuations may be large.

Despite the limitation of the data, our study has yielded reasonable magnitudes for the elastic properties of the aquifer. Carefully controlled experiments in the future (when measurements of gravity, barometric pressures, and fluid pressures are taken over long periods with packed-off as well as open wells) would be desirable. Indeed, in a well where earth tide and barometric responses are simultaneously seen, c_p could be estimated from barometric data and then used to estimate $\delta\sigma_m$ for the earth tide problem. This $\delta\sigma_m$ could give a very realistic estimate of the stresses generated within the earth owing to the transit of celestial objects.

NOTATION

- a radius of earth [L].
- c_m bulk compressibility of porous medium with reference to mean principal stress [LT^2/M].
- c_p pore volume compressibility [LT^2/M].
- c_t total compressibility [LT^2/M].
- c_w compressibility of water [LT^2/M].
- D damping coefficient.

- E_L lag coefficient.
- E_s modulus of compression of soil skeleton confined in situ [M/LT^2].
- E_w bulk modulus of elasticity of water [M/LT^2].
- g acceleration due to gravity [L/T^2].
- δh change in water level in well [L].
- H thickness of aquifer or length of well casing [L].
- k harmonic number.
- K bulk modulus [M/LT^2].
- L length of hydrograph record [T].
- m_v coefficient of volume change [LT^2/M].
- p fluid pressure [M/LT^2].
- δp_A change in barometric pressure [M/LT^2].
- δp_{well} change in fluid pressure in well [M/LT^2].
- r_0 effective radius [L].
- r_w radius of well [L].
- S storage coefficient of aquifer.
- S_s specific storage [$1/L$].
- S_{sw} storage coefficient of well [L^3/L].
- T transmissivity of aquifer [L^2/T].
- T period of tidal component.
- V volume [L^3].
- V_0 initial volume [L^3].
- V_w volume of water [L^3].
- δV_w change in water volume [L^3].
- w angular speed.
- w_2 tidal potential [L^2/T^2].
- γ_w unit weight of water [M/L^2T^2].
- Δ dilatation of the porous medium.
- Δ_t tidal dilatation of Bredehoeft [1967].
- Δ_w dilatation of water.
- Δ_{drained} ratio of volume of water drained to a total volume.
- ε_v vertical strain.
- $\delta\sigma_m'$ change in mean principal total stress [M/LT^2].
- $\delta\sigma_e'$ change in mean principal effective stress [M/LT^2].
- $\delta\sigma_v'$ change in mean principal vertical stress [M/LT^2].
- ϕ porosity.

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REFERENCES

- Arditty, P. C., H. J. Ramey, Jr., and A. M. Nur, Response of a closed well reservoir system to stress induced by earth tides, paper presented at the Society of Petroleum Engineers/AIME, 53rd Annual Meeting, Houston, Tex., Oct. 1978.
- Blanchard, F. G., and P. Byerly, A study of a well gauge as a seismograph, *Bull. Seismol. Soc. Am.*, 25, 313-321, 1935.
- Boydvarsson, G., Confined fluids as strain meters, *J. Geophys. Res.*, 75(14), 2711-2718, 1970.
- Bredehoeft, J. D., Response of well-aquifer systems to earth tides, *J.*

- Geophys. Res.*, 72(12), 3075–3087, 1967.
- Cooper, H. H., Jr., J. D. Bredehoef, and I. S. Papadopoulos, Response of a finite diameter well to an instantaneous change of water, *Water Resour. Res.*, 3(1), 263–269, 1967.
- Dale, R. H., Relationship of groundwater tides to ocean tides: A digital simulation model, Ph.D. dissertation, 150 pp., Univ. of Hawaii, Honolulu, 1974.
- Harrison, J. C., *New Computer Programs for the Calculation of Earth Tides*, Cooperative Institute for Research in Environmental Sciences, University of Colorado, Boulder, 1971.
- Kanehiro, B. Y., A hydrogeologic study of the Kiholo to Puako area on the Island of Hawaii, M.S. thesis, 147 pp., Univ. of Hawaii, Honolulu, 1977.
- Kanehiro, B. Y., The response of aquifers to earth tides, M.S. thesis, Univ. of Calif., Berkeley, 1979.
- Lambe, T. W., and R. V. Whitman, *Soil Mechanics*, John Wiley, New York, 1969.
- Love, A. E. H., *Some Problems of Geodynamics*, 180 pp., Cambridge University Press, New York, 1911.
- Marine, I. W., Water level fluctuations due to earth tides in a well pumping from slightly fractured crystalline rocks, *Water Resour. Res.*, 11(1), 165–173, 1975.
- Melchior, P., Die gezeiten in unterirdischen flussigkeiten, *Erdoel Kohle*, 13, 312–317, 1960.
- Melchior, P., A. Sterling, and A. Wery, Effets de dilatations ambiguës due aux marées terrestres observés sous forme de variations de niveau dans un puits, à Basecles (Hainaut), *Commun. Observ. R. Belgique*, no. 224, 12 pp., 1964.
- Narasimhan, T. N., and B. Y. Kanehiro, A note on the meaning of storage coefficient, *Water Resour. Res.*, 16(2), 423–429, 1980.
- Narasimhan, T. N., and P. A. Witherspoon, Reservoir evaluation, tests on RRGE 1 and RRGE 2, Raft River Geothermal Project, Idaho, Rep. LBL-5958, Lawrence Berkeley Lab., Berkeley, Calif., 1977.
- Narasimhan, T. N., and P. A. Witherspoon, Numerical model for saturated unsaturated flow in deformable porous media, 3, Applications, *Water Resour. Res.*, 14(6), 1017–1034, 1978.
- Narasimhan, T. N., D. G. McEdwards, and P. A. Witherspoon, Results of reservoir evaluation tests, 1976, East Mesa, California, Rep. LBL-6369, Lawrence Berkeley Lab., Berkeley, Calif., 1977.
- Narasimhan, T. N., P. A. Witherspoon, and A. L. Edwards, Numerical model for saturated unsaturated flow in deformable porous media, 2, The algorithm, *Water Resour. Res.*, 14(2), 255–261, 1978.
- Robinson, E. S., and R. T. Bell, Tides in confined well-aquifer systems, *J. Geophys. Res.*, 76(8), 1857–1869, 1971.
- Stacey, F. D., *Physics of the Earth*, 2nd ed., 414 pp., John Wiley, New York, 1969.
- Takeuchi, H., On the earth tide of the compressible earth of variable density and elasticity, *Eos Trans. AGU*, 31, 651–689, 1950.
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