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# Effect of Storm Scale on Surface Runoff Volume

P. C. D. MILLY

*Department of Civil Engineering and Operations Research, Water Resources Program, Princeton University, Princeton, New Jersey*

P. S. EAGLESON

*Department of Civil Engineering, Massachusetts Institute of Technology, Cambridge*

Dynamic hydrologic models of areas that are potentially larger than characteristic storm sizes must give explicit consideration to the effect of storm size on the rainfall-runoff process. The local response of an element of the modeled area can be parametrized in terms of proportion of saturated and impermeable area and infiltration parameters of the unsaturated permeable areas. When a very simple spatial description of the storm depth and duration is provided, it is possible to integrate the local response over the entire area to obtain the average, or lumped, input-output behavior of the large area on a volume basis. Where infiltration-excess runoff is significant, the rainfall-runoff response of a large area is extremely sensitive to the storm size, with a fixed volume of precipitation producing more runoff when it is concentrated over smaller areas. Where saturated or impermeable source areas provide most of the surface runoff, this scale effect on runoff volume is absent. Whenever the effect is significant, it becomes important for rainfall-runoff models to include not only the rainfall volume but also some description of its areal variability, as forcing variables.

## INTRODUCTION

One goal of physical hydrology is to describe hydrologic processes at catchment scale by means of models derived from the underlying physics. The first approximation is to represent a catchment as a homogeneous one-dimensional system subjected to uniform atmospheric forcing, i.e., precipitation and potential evaporation. Due to the enormous spatial variability of physical parameters and forcing functions in nature, such a description does not appear to be accurate in many cases. Several studies have been conducted to examine the effect of soil spatial variability on physical processes related to soil water [*Peck et al.*, 1977; *Smith and Hebbert*, 1979; *Sharma and Luxmoore*, 1979; *Freeze*, 1980; *Milly and Eagleson*, 1982, 1987; *Dagan and Bresler*, 1983; *Bresler and Dagan*, 1983*a*, *b*; *Yeh et al.*, 1985*a*, *b*, *c*]. The effect of spatial variability of storm properties has been less intensively studied, since the soil is usually much more variable than the precipitation on small catchments. *Freeze* [1980] considered local variability of precipitation at the hillslope scale. *Milly and Eagleson* [1982] used Monte Carlo methods to examine the effect of storm size on the rainfall-runoff relation for relatively large areas.

The prediction of general circulation of the atmosphere by numerical climate models requires descriptions of land surface boundary conditions at model grid scales, typically on the order of hundreds of kilometers. At such scales the importance of large-scale spatial heterogeneity of precipitation events takes on real significance, since characteristic storm scales may be much smaller than these. In this paper we continue the work of *Milly and Eagleson* [1982] describing the effects of storm size on surface runoff volumes in situations where the storm size is not necessarily large compared to the modeled area. We consider runoff production by the classical infiltration-excess mechanism and by saturated and impermeable source areas, and we develop exact expressions for the areal average response.



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In the following sections we describe a simple conceptual model of surface runoff from small elements of land areas, postulate three simple descriptions of large-scale variation of storm depth and duration, derive expressions for the areal average surface runoff volume, and present computed results. We discuss briefly the implications for climate modeling.

## SURFACE RUNOFF AT A POINT

In response to storm precipitation there is a quick response of streamflow due to a number of physical mechanisms [*Beven*, 1983]. Where and when the precipitation rate exceeds the infiltration capacity of previously unsaturated soil, the excess is available for production of overland flow. Where the soil profile is initially saturated, as in the case of a seepage face or swamp, and where the surface is impermeable, all precipitation is immediately available for overland flow. Either mechanism may be accelerated by topographically induced accumulation of lateral moisture flow.

For our analysis of the effect of storm size a relatively simple model of runoff generation seems justified. We divide the modeled area into two subareas. The first subarea, having proportion $M^S$ of the total area, includes water surfaces, impermeable areas, saturated land areas, and areas that saturate rapidly during significant storm events; all water falling on this subarea is presumed to produce surface runoff. The second subarea, having proportion $M^I(=1 - M^S)$ of the total area, produces runoff only when sufficient infiltration has occurred to reduce infiltration capacity below the rainfall rate. For the sake of expedience we treat $M^S$ and $M^I$ as constants for a given storm, although we recognize that the division between subareas can vary during the course of a storm. In the development that follows we shall assume that each element of modeled area has the same partitioning between source area types as implied by $M^S$ and $M^I$.

We denote by $Y$ the volume of total storm runoff per unit modeled area. Then

$$Y = M^I Y^I + M^S Y^S$$

or

$$Y = M^I Y^I + (1 - M^I) Y^S \tag{1}$$

where $Y^I$ is the depth of infiltration-excess runoff produced at a point, and $Y^S$ is simply equal to the local storm depth $h$,

$$Y^S = h \tag{2}$$

In principle, an approach to runoff based on the concept of infiltration capacity requires that the time dependence of the precipitation intensity $p$ be known. When normalized by storm duration and total storm depth, average plots of cumulative rain against time follow characteristic patterns determined by the type of precipitation process [*Eagleson*, 1970]. Superimposed upon this intensity variation is a higher-frequency random component associated with the passage of internal storm features of varying scales [*Gupta and Waymire*, 1979]. In this study we are concerned primarily with spatial variability, and therefore we feel justified in adopting a temporally constant rainfall intensity to describe each element of area of the land surface. Furthermore, it is arguable that surface depression and detention storage of water provide a filter that effectively eliminates the higher-frequency fluctuations. It does not appear that the constant intensity approximation will significantly distort our results concerning spatial variability.

In order to determine $Y^I$ we require a description of the infiltration capacity. *Philip* [1957] presented an approximate solution for infiltration into a uniformly wetted homogeneous soil when an unlimited supply of free water is applied to the surface. His two-term algebraic infiltration equation is

$$F_i = S_i t^{1/2} + A_0 t \tag{3}$$

in which $F_i$ is the cumulative depth of infiltered water when time $t$ has elapsed. The parameter $S_i$ is the sorptivity, and $A_0$ is the infiltration rate at large time. By definition, the infiltration capacity $f_i^*$ for this problem is identical to the actual infiltration rate, which is the rate of change of $F_i$,

$$f_i^* = \frac{1}{2} S_i t^{-1/2} + A_0 \tag{4}$$

Elimination of $t$ between (3) and (4) yields

$$f_i^* = A_0\{1 + [-1 + (1 + 4A_0F_i/S_i^2)^{1/2}]^{-1}\} \tag{5}$$

For the problem of rainfall infiltration the water supply rate is finite, and neither (3) nor (4) is directly applicable. They may be applied, however, in conjunction with the time compression approximation [*Reeves and Miller*, 1975] to model rainfall infiltration. Under time compression the infiltration capacity is dependent only upon the cumulative depth of infiltration at any time. This means that (5) is assumed valid regardless of the infiltration history. Equivalently, the increase in $F_i$ with time under conditions of potential infiltration ($p$ in excess of infiltration capacity) has a unique shape, such that (3) and (4) apply when the time variable is shifted by some appropriate amount $t'$. Thus (3) becomes

$$F_i = S_i(t - t')^{1/2} + A_0(t - t') \tag{6}$$

If storm intensity and duration are sufficiently great, infiltration-excess runoff begins at the so-called "ponding" time $t_0$, defined as the time when the infiltration capacity $f_i^*$ has fallen to the precipitation rate $p$. The cumulative infiltration at that time is $pt_0$. Substitution of this information into (5) yields an expression for $t_0$,

$$t_0 = \frac{S_i^2(2p - A_0)}{4p(p - A_0)^2} \qquad p > A_0 \tag{7}$$

After time $t_0$, (6) applies. By forcing (6) to give $F_i$ equal to $pt_0$ at time $t_0$, and noting that the time derivative of (6) should give $f_i^*$ equal to $p$ at the same time, we find an expression for $t'$,

$$t' = \frac{S_i^2}{4p(p - A_0)} \qquad p > A_0 \tag{8}$$

We find $Y^I$ by subtracting total storm infiltration from total storm depth. For a storm of duration $t_r$ this leads to

$$Y^I = 0 \qquad p \le A_0 \quad \text{or} \quad t_r \le \frac{S_i^2(2p - A_0)}{4p(p - A_0)^2}$$

$$Y^I = pt_r - A_0\left[t_r - \frac{S_i^2}{4p(p - A_0)}\right] - S_i\left[t_r - \frac{S_i^2}{4p(p - A_0)}\right]^{1/2} \tag{9}$$

$$p > A_0 \quad \text{and} \quad t_r > \frac{S_i^2(2p - A_0)}{4p(p - A_0)^2}$$

## STORM DESCRIPTION

The spatial and temporal structure of a precipitation event is complex and depends upon storm type. Given our neglect of time variations of intensity, there remains simply the problem of specifying the nature of the spatial variations of intensity $p$ and duration $t_r$. One parameter whose large-scale spatial structure has been intensively studied is the total storm depth, a variable for which numerous records are available. *Eagleson* [1970] shows the decay of rainfall with normalized distance from the storm center for storms of various types. The decrease with distance is most abrupt in convective storms and more gradual in cyclones. The relation given by *Chow* [1954] for small extratropical cyclones is

$$h_A = h_0(1 + A/A_c)^{-1} \tag{10}$$

where $h_A$ is the average storm depth within a region of area $A$ enclosed by an isohyet, $h_0$ is the maximum storm depth, and $A_c$ is a characteristic storm area (520 km$^2$). We shall employ this relation for storms of varying characteristic sizes $A_c$. If we take the isohyets to be circular, then we may use (10) to derive an expression for point rainfall depth $h$ as a function of distance $r$ from the storm center,

$$h = h_0[1 + (r/r_0)^2]^{-2} \tag{11}$$

in which

$$r_0 = (A_c/\pi)^{1/2} \tag{12}$$

In the case of convective precipitation the reduction of $h$ with $r$ is much sharper than that suggested by (11), so we shall also consider the much simpler model

$$h = h_0 \qquad r < r_0$$
$$h = 0 \qquad r \ge r_0 \tag{13}$$

In both (11) and (13), $r_0$ is a characteristic storm radius. We expect that (13) is more appropriate for small storms and (11) for large storms.

The spatial variation of $h$ can naturally be decomposed into variations of $p$ and of $t_r$, whereupon (11) may be written

$$pt_r = p_0t_{r0}[1 + (r/r_0)^2]^{-2} \tag{14}$$

where $p_0$ and $t_{r0}$ are the intensity and duration at the storm center. For model (13) it seems reasonable to consider $t_r$ as constant. For (11) the reduction of storm depth at the edges of the storm is due to the combination of lower intensities and a shorter period of rain. In the development to follow we shall examine the two extreme cases where variability of $h$ is attributed entirely to $t_r$ ($p = p_0$) or entirely to $p$ ($t_r = t_{r0}$).

## AREAL AVERAGE RUNOFF

Consider an area of land (for example, a catchment) that is not necessarily small relative to the size of a particular storm. For simplicity we represent the land area by a circular region of radius $R$ and consider that the storm and this area are concentric. Then the areal average of $Y$ on the land area, denoted by $\langle Y \rangle$, may be calculated according to

$$\langle Y \rangle = \frac{1}{\pi R^2} \int_0^{2\pi} \int_0^R Y(r, \theta) r \, dr \, d\theta \tag{15}$$

where $r$ and $\theta$ are used as polar coordinates. Since we are examining the effects of storm variability only, we treat the soil and catchment parameters as constants. Thus the only remaining spatial dependence of $Y$ is due to the dependence of $p$ or $t_r$ on radial distance, and (15) may be expressed as

$$\langle Y \rangle = \frac{2}{R^2} \int_0^R Y[p(r), t_r(r)] r \, dr \tag{16}$$

Using (1) and (2), we find

$$\langle Y \rangle = (1 - M^I)\langle h \rangle + M^I \langle Y^I \rangle \tag{17}$$

where $\langle h \rangle$ is the average storm depth inside the modeled area and $\langle Y^I \rangle$ is the average value of $Y^I$,

$$\langle Y^I \rangle = \frac{2}{R^2} \int_0^R Y^I r \, dr \tag{18}$$

The average storm depth for (14) is

$$\langle h \rangle = h_0[1 + (R/r_0)^2]^{-1} \tag{19}$$

and for (13) it is

$$\langle h \rangle = h_0 \min [1, r_0^2/R^2] \tag{20}$$

In the derivation of $\langle Y^I \rangle$ we consider the following three cases: case 1A, $h$ given by (14), $t_r$ constant; case 1B, $h$ given by (14), $p$ constant; and case 2, $h$ given by (13), $p$ and $t_r$ constant for $r < r_0$.

*Case 1A: h Given by (14), $t_r$ Constant*

For constant $t_r$, which we denote here by $t_{r0}$, (14) implies

$$p = p_0[1 + (r/r_0)^2]^{-2} \tag{21}$$

If $p_0$ and $t_{r0}$ are sufficiently small, then $Y^I$ is zero at the center and hence everywhere,

$$\langle Y^I \rangle = 0 \qquad p_0 \le A_0 \quad \text{or} \quad t_{r0} \le \frac{S_i^2(2p_0 - A_0)}{4p_0(p_0 - A_0)^2} \tag{22}$$

It will greatly simplify notation if we introduce the following dimensionless quantities:

$$\alpha = A_0/p_0 \tag{23}$$

$$\sigma = S_i/2p_0 t_{r0}^{1/2} \tag{24}$$

Then (22) may be written

$$\langle Y^I \rangle = 0 \qquad \alpha \ge 1 \quad \text{or} \quad \sigma^2(2 - \alpha) \ge (1 - \alpha)^2 \tag{25}$$

If $p_0$ and $t_{r0}$ are sufficiently large, then $Y^I$ will not be zero at the center. However, $Y^I$ will decrease to zero at some distance $R^I$ from the storm center, where the storm intensity is defined as $p^I$. $R^I$ may be either less than or greater than $R$, the radius of the modeled area. Reference to (9) reveals that $p^I$ obeys

$$p^I/p_0 > \alpha \tag{26}$$

$$(p^I/p_0)[(p^I/p_0) - \alpha]^2 = \sigma^2[2(p^I/p_0) - \alpha] \tag{27}$$

It can be shown that (27) has three distinct real roots $p^I$, and that only the largest of these obeys (26); together these equations determine $p^I$. Given $p^I$, we find $R^I$ according to

$$R^I = r_0[(p_0/p^I)^{1/2} - 1]^{1/2} \tag{28}$$

We may now evaluate (18) by substituting the nonzero expression for $Y^I$ from (9) and integrating from zero to $R^I$ or $R$, whichever is smaller. This leads to

$$\begin{aligned}\langle Y^I \rangle = {} & (p_0 t_{r0} r_0^2/R^2)\{\rho^2/(1+\rho^2) - \alpha\rho^2 \\ & - \sigma^2[(1+1/\alpha)\rho^2 + \rho^4 + \tfrac{1}{3}\rho^6] \\ & + \sigma^2\alpha^{-3/2}[\mathrm{Arth}[\alpha^{1/2}(1+\rho^2)] - \mathrm{Arth}[\alpha^{1/2}]] \\ & - 2\sigma\alpha^{-1/2} J[\sigma/\alpha, \alpha^{1/2}(1+\rho^2)] + 2\sigma\alpha^{-1/2}) J[\sigma/\alpha, \alpha^{1/2}]\} \\ & \alpha < 1 \quad \text{and} \quad \sigma^2(2-\alpha) < (1-\alpha)^2\end{aligned} \tag{29}$$

in which the dimensionless quantity $\rho$ is defined by

$$\rho = \min [R^I, R]/r_0 \tag{30}$$

Arth( ) is the inverse hyperbolic tangent, and $J[\ ,\ ]$ is defined by

$$J[a, b] = \int_0^b \left[\frac{1 - z^2 - a^2 z^4}{1 - z^2}\right]^{1/2} dz \tag{31}$$

*Case 1B: h given by (14), p Constant*

For $p$ everywhere equal to $p_0$, (14) implies

$$t_r = t_{r0}[1 + (r/r_0)^2]^{-2} \tag{32}$$

As in case 1A, there is no runoff anywhere if there is none at the storm center,

$$\langle Y^I \rangle = 0 \qquad \alpha \ge 1 \quad \text{or} \quad \sigma^2(2 - \alpha) \ge (1 - \alpha)^2 \tag{33}$$

For the case where $Y^I$ is nonzero at the center we find

$$R^I = r_0\{[(1 - \alpha)^2/\sigma^2(2 - \alpha)]^{1/2} - 1\}^{1/2} \tag{34}$$

Integration of (18) yields

$$\begin{aligned}\langle Y^I \rangle = {} & (p_0 t_{r0} r_0^2/R^2)\{(1-\alpha)\rho^2/(1+\rho^2) + \alpha\sigma^2\rho^2/(1-\alpha) \\ & - 2\sigma[1 - \sigma^2(1+\rho^2)^2/(1-\alpha)]^{1/2} + 2\sigma[1 - \sigma^2/(1-\alpha)]^{1/2} \\ & + 2\sigma \,\mathrm{Arth}\, [[1 - \sigma^2(1+\rho^2)^2/(1-\alpha)]^{1/2}] \\ & - 2\sigma \,\mathrm{Arth}\, [[1 - \sigma^2/(1-\alpha)]^{1/2}]\}\end{aligned} \tag{35}$$

where $\rho$ is given by (30).

*Case 2: h Given by (13), p and $t_r$ Constant for $r < r_0$*

For this case the areal averages follow directly from (9), (14), and (18),

$$\langle Y^I \rangle = 0 \qquad \alpha \ge 1 \quad \text{or} \quad \sigma^2(2 - \alpha) \ge (1 - \alpha)^2 \tag{36}$$

$$\begin{aligned}\langle Y^I \rangle = {} & [p_0 t_{r0} \min (1, r_0^2/R^2)] \\ & \cdot \{1 - \alpha + \alpha\sigma^2/(1-\alpha) - 2\sigma[1 - \sigma^2/(1-\alpha)]^{1/2}\} \\ & \alpha < 1 \quad \text{and} \quad \sigma^2(2-\alpha) < (1-\alpha)^2\end{aligned} \tag{37}$$

## RESULTS

We display the results in dimensionless form as normalized infiltration-excess runoff $\langle Y^I \rangle / \langle h \rangle$ versus relative storm size

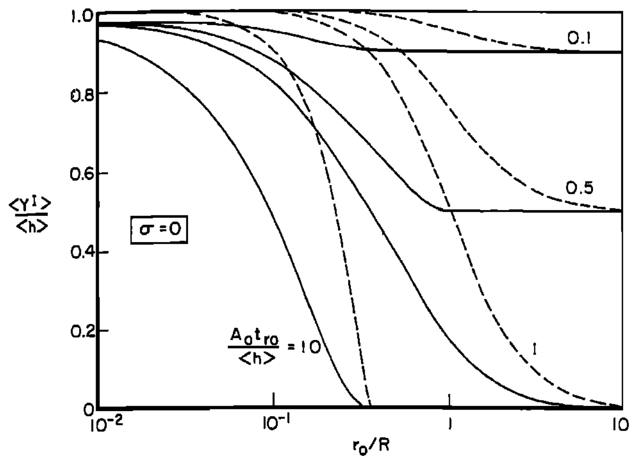


Fig. 1. Normalized runoff due to infiltration-excess $\langle Y^I \rangle/\langle h \rangle$ versus relative storm size $r_0/R$. Sorptivity equal to zero. Constant $t_r$ (solid lines) and constant $p$ (dashed lines).

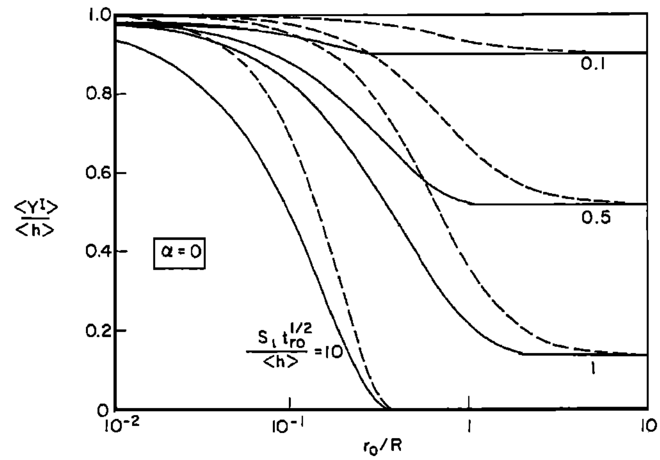


Fig. 3. Normalized runoff due to infiltration-excess $\langle Y^I \rangle/\langle h \rangle$ versus relative storm size $r_0/R$. $A_0$ equal to zero. Constant $t_r$ (solid lines) and constant $p$ (dashed lines).

$r_0/R$ for various constant values of the parameters

$$\frac{\alpha h_0}{\langle h \rangle} = \frac{A_0 t_{r0}}{\langle h \rangle}$$

$$\frac{2\sigma h_0}{\langle h \rangle} = \frac{S_i t_{r0}^{1/2}}{\langle h \rangle}$$

Referring to (3), we see that $A_0 t_{r0}$ represents the potential infiltration during a storm of long duration or in a soil such as coarse sand where the sorptivity $S_i$ may be neglected. In contrast, $S_i t_{r0}^{1/2}$ represents the potential infiltration during a storm of short duration or in a soil such as a heavy clay in which the effect of gravity on infiltration is negligibly small.

Figure 1 shows the normalized runoff for $t_r$ = const (solid lines) and for $p$ = const (dashed lines) as a function of relative storm size under the condition that $\sigma = 0$. This represents the simple case in which infiltration capacity is effectively constant through time such as might occur with a coarse sand. Note that the parameter $A_0 t_{r0}/\langle h \rangle$ allows tracing along a single curve the effect of nonuniform spatial distribution of storms of constant catchment average depth. For large relative storm size the rainfall becomes spatially uniform, and the curves approach the horizontal asymptotes

$$\frac{\langle Y^I \rangle}{\langle h \rangle} = 1 - \frac{A_0 t_{r0}}{\langle h \rangle}$$

suggested by (9), since in this limiting case, $A_0 t_{r0} \equiv \langle A_0 t_r \rangle$. For small relative storm size, most rainfall is forced onto a small area of finite infiltration capacity, causing nearly all water to run off. An enormous range of storm sizes lies between these two limits, reflecting the sensitivity of the rainfall-runoff process to the horizontal length scale of the storm within that range. Rainfall spread increasingly widely over an area produces increasingly less infiltration-excess runoff. For a variable storm duration $t_r$ (dashed lines) the scale effect is more pronounced over a narrower range of scales than for a variable intensity (solid lines); the true behavior is probably intermediate between those implied by these curves.

Figure 2 shows the equivalent results for constant $p$ and $t_r$. Understandably, the sensitivity of runoff to storm size is even greater in this case than in those of Figure 1, since the storm has a sharp boundary.

Figures 3 and 4 display similar results for the sorptivity-controlled situation where $\alpha$ is zero, which should represent the behavior of a heavy clay. The results are very similar to those for gravity-controlled infiltration in Figures 1 and 2. For large relative storm size the curves approach the horizontal

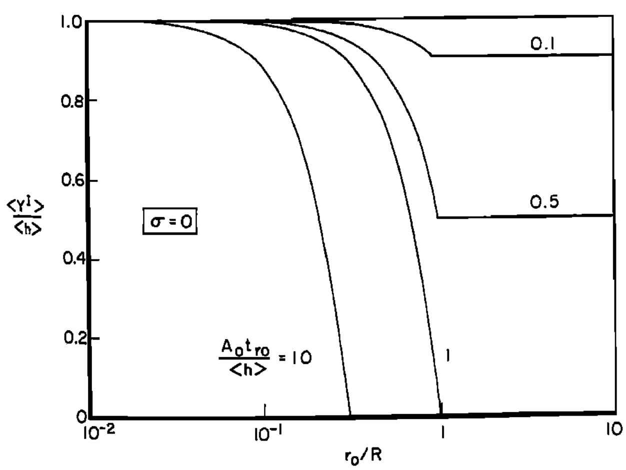


Fig. 2. Normalized runoff due to infiltration-excess $\langle Y^I \rangle/\langle h \rangle$ versus relative storm size $r_0/R$. Sorptivity equal to zero. Constant $p$ and $t_r$.

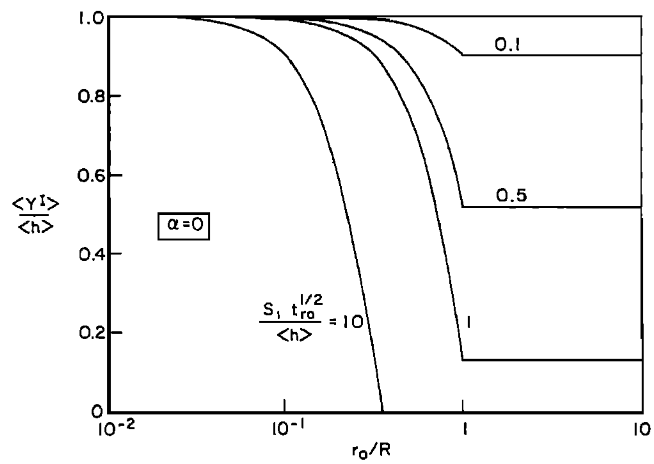


Fig. 4. Normalized runoff due to infiltration-excess $\langle Y^I \rangle/\langle h \rangle$ versus relative storm size $r_0/R$. $A_0$ equal to zero. Constant $p$ and $t_r$.

asymptotes implied by (9),

$$\frac{\langle Y^I \rangle}{\langle h \rangle} = 1 - \frac{S_i t_{r0}{}^{1/2}}{\langle h \rangle}\left[1 - \frac{1}{4}\left(\frac{S_i t_{r0}{}^{1/2}}{\langle h \rangle}\right)^2\right]^{1/2}$$

A comparison of the results for limiting cases $\alpha = 0$ and $\sigma = 0$ reveals that the two families of runoff curves are virtually identical in cases where storm duration $t_r$ is spatially constant. This means that the form of the relation between normalized runoff and relative storm size is effectively independent of whether sorptivity or gravity dominates the infiltration process. For $t_r$ spatially variable the form of the relation between runoff and storm size is influenced by the dominant infiltration mechanism but not to a great extent.

According to (2), there is no scale-dependence of the $\langle Y^S \rangle$ component of $\langle Y \rangle / \langle h \rangle$. From (1) it is then clear that the scale effect is absent if $M^I$ is zero and becomes increasingly important as $M^I$ increases.

## DISCUSSION

We have shown that the areal distribution of a given volume of precipitation strongly affects the amount of direct runoff due to the infiltration-excess runoff mechanism. Although we considered only a simple, deterministic variation, the results are qualitatively of more general applicability. In general, it may be stated that spatial variability of precipitation induces increased surface runoff compared to a uniform rainfall. In this sense the effect of rain variability is very much like the more thoroughly studied effect of soil variability. The consequences for modeling are somewhat different, however, as rainfall variability is event-dependent, whereas soil variability is essentially static. The implication is that a hydrologic model representing a sufficiently large area must incorporate some representation of areal storm variability. The grid scale of general circulation models ($10^5$ km$^2$) is certainly large enough to require this. If one instead ignores the areal variability, the potential exists for drastic underestimation of surface runoff volume. It is possible that considerably smaller areas (e.g., 10 km$^2$) are also large enough to require areal distributions, given the significant deterministic and stochastic variations in rainfall that often occur on such scales due to the cellular nature of many storms. These results underline the importance of the development of models describing areal distributions of storm depth and duration.

## NOTATION

- $A_c$ characteristic storm area.
- $A_0$ steady infiltration capacity at large time.
- $\alpha$ dimensionless quantity $A_0/p_0$.
- $F_i$ cumulative depth of infiltration.
- $f_i^*$ infiltration capacity.
- $h$ storm depth.
- $h_0$ maximum storm depth.
- $M^I$ proportion of modeled area producing infiltration-excess runoff.
- $M^S$ proportion of modeled area converting all rain to runoff.
- $p$ precipitation intensity.
- $p_0$ maximum precipitation intensity.
- $p^I$ precipitation intensity at which $t_0$ equals $t_r$.
- $R$ radius of modeled area.
- $R^I$ radial distance at which $Y^I$ goes to zero.
- $r$ radial distance from center of storm and modeled area.
- $r_0$ characteristic storm radius.
- $\rho$ dimensionless radius defined by (29).
- $S_i$ sorptivity parameter in Philip's infiltration equation.
- $\sigma$ dimensionless quantity $S_i/2p_0t_{r0}{}^{1/2}$.
- $t$ time.
- $t'$ time shift in infiltration equation.
- $t_r$ storm duration.
- $t_0$ time at which infiltration-excess runoff begins.
- $t_{r0}$ value of $t_r$ at center of storm.
- $Y$ total runoff depth.
- $Y^I$ infiltration-excess runoff depth.
- $Y^S$ runoff depth from saturated and impermeable areas.

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P. S. Eagleson, Department of Civil Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139.

P. C. D. Milly, Department of Civil Engineering and Operations Research, Water Resources Program, Princeton University, Princeton, NJ 08544.

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