DISCUSSION

H. R. Leach (5536 30th Place, Washington, D. C.)—The author apparently neglects to include the initial losses which occur prior to the beginning of runoff. These amounted in Run 10 to 0.35 inch and in Run 22 to 0.14 inch, accounted for partly by infiltration during the period until runoff started and partly by the filling of permanently retained depression-storage. In the first instance this is over one-quarter of the total rainfall and, were the plot-results directly applicable to even a small area, would materially reduce the peak-rate of runoff for any concentration-period exceeding 37 minutes.

Relative to the inquiry in the conclusions as to whether high intensity increases the rate of infiltration, it can be said that there is nothing in the few runs thus far made at Lincoln to indicate that such is the case, an intensity of 2.60 inches on Plot 2 having given an infiltration-rate of approximately 1.15 inches per hour in contrast to the 1.85 inches per hour obtained for a 2.5-inch intensity.

The results of plot-tests must be used with caution when applied to larger areas. One of the greatest problems in connection with such tests relates to their use in predicting the performance of natural watersheds of areas even so small as two or three acres when subjected to natural storm-rainfall.

SOME CHANNEL-STOREAGE STUDIES AND THEIR APPLICATION TO THE DETERMINATION OF INFILTRATION

W. B. Langbein

Results of studies of recent major floods have emphasized the practical value of analyses of the relation between rainfall and resultant runoff. Analyses that are directed toward appraising the respective values of the soil and the river-channel as agencies for disposing of potential flood-waters are especially pertinent.

If evaporation-losses from non-infiltrated water may be neglected as inconsequential, as during prolonged storms of winter, the difference between rainfall and the associated runoff may be ascribed to such edaphic influences represented by infiltration, interception, and surface-storage. The problem in the analyses is therefore one of correlating rainfall and resultant runoff. The procedures described in this paper resulted from studies of the rainfall and runoff relations of the Ohio River flood of January 1937 [see 1 of References at end of paper].

Methods of computing mean areal precipitation are generally well known and will not be discussed here. The accuracy of the computed mean areal precipitation, by whatever method chosen, is dependent upon the adequacy of the number of rain-gages per unit of area and their distribution with respect to the basin in question to define the geographical variation of precipitation. The segregation of daily observations of precipitation into separate storm-periods is generally not difficult. However, such separate storm-periods as may be recognized in the record of precipitation, frequently may be identified only with great difficulty upon the hydrographs of resultant stream-flow, largely due to the modulating effects of channel-storage.

Channel storage—The identity of irregular and seemingly erratic contributions of rainfall to surface-runoff tend to become increasingly obscured by channel-storage effects as they flow downstream through natural channels. The resultant hydrographs are frequently so smooth that they afford little, if any, indication of the details of the distribution of rainfall that produced the stream-flow, largely due to the modulating effects of channel-storage.

Attempts have been made, therefore, to adjust hydrographs of stream-flow for the effects of channel-storage, and so to compute the hydrograph of surface-inflow into the channel-system. The rate of inflow into channels at a given time above a given point of measurement of discharge, obviously is equal to the rate of outflow plus or minus the rate of change of total storage in the upstream channel-system. Knowing the outflow-discharge the problem becomes essentially the computation of the total storage in the upstream channels at appropriate intervals of time.

The volume of water in a channel-system is assumed to be directly related to the prevailing stages. However, in those streams where definite relations exist between stage and discharge, the rate of discharge provides a more convenient index to the volume of channel-storage. The recession-limb of a discharge hydrograph represents water draining from the channel-system including the river-flood plains and the volume in channel-storage (above base or ground-water flow) may be related to the rate of discharge from such storage [2].
In small basins, the discharge at the outlet point may serve as a satisfactory index of the stages prevailing upstream. However, in large basins, particularly those of drainage-area over 500 square miles, one gage at the outlet of the basin will not generally serve as a sufficient index. For example, at the beginning of a storm, the gage at the outlet may show but negligible rise, yet the channels upstream may be considerably swollen, and moreover, in the larger basins, after the storm has ended, the lower may be at peak and yet the stations upstream may have long passed their peak. Furthermore, floods over large basins are not of uniform intensity. To evaluate these possibilities adequately, it is essential to make use of upstream gages, by using each gage only as an index of channel-storage in the area intervening between it and other gages upstream. Naturally the accuracy obtainable largely depends on the adequacy of the number of gaging stations available for this purpose. Through the use of upstream gages, weight may be given to floods which affect only a part of the basin or to the variable flood-intensities over the basin.

In the preparation of the discharge-channel-storage relationship in a given basin, study must be made of past floods to obtain recession-hydrographs that are free from contemporaneous inflow from surface-water. It has been found that complete recession-hydrographs meeting this requirement are rare. This experience led to the adoption of the following procedure for determining a normal recession-graph or graph free from complications of inflow. When two or more days of apparently normal recession have been found, one day's discharge has been plotted as ordinate against the following day's discharge as abscissa on rectangular coordinates, as illustrated on Figure 1. (Base or ground-water discharge must of course be deducted.) Lines have been drawn connecting points on the same recession-graph. These points and lines were then used as a basis.
Second-feet per square mile

FIG. 2—TYPICAL RECESSION-CURVES

for drawing a curve which must pass through the origin, which curve has been termed the recession-curve. In drawing the recession-curve, greater weight is given to these points which show the most rapid rate of draining out, that is, those which plot to the left on Figure 1 and which show a tendency to define a curve. Care is exercised, however, to disregard those singular points which plot so far to the left as to be indicative of drainage from partial-area flood-rises.

The shape of the recession-curve is indicative of the channel-storage characteristics of the particular basin. A straight-line recession-curve indicates that the recession-hydrograph is of exponential form, \( q = ae^{-bt} \) where \( a \) and \( b \) are constants, \( q \) is discharge, and \( t \) is time, in which event the curve of the discharge-channel-storage relation is a straight line at a 45° slope on logarithmic paper, and the volume of channel-storage bears a constant ratio to the discharge-rate. The recession-curves of small upstream basins frequently are steep at high discharge-rates but rapidly flatten as they approach the origin (see curve A of Fig. 2). This characteristic of these curves is due to the high velocities peculiar to these streams and efficient channel-conveyance at high rates of discharge incident to steep slopes, whereas at low rates of discharge there are numerous pools to be drained eliminating the influence of slope and with channel-conveyance relatively poor provided these pools are not drowned out by a high base or ground-water flow. The recession-curves of large basins, on the other hand, appear to be steeper at low discharge-rates than at higher discharge-rates. The upper end of the curve cannot exceed a limiting 45° slope (see curve B of Fig. 2). The lower reaches of large streams have wide overflow areas which provide large volumes of channel-storage but do not add proportionately to the rate of discharge inasmuch as overflow areas do not flow at efficient rates of discharge. Large rivers, however, flow with relatively greater velocity at low discharge-rates when the river is
The recession-curve may be used to compute the actual normal recession-hydrograph from storage of surface-water runoff in the channel-system, from which the discharge-channel-storage relation may then be computed and plotted on log paper. Figure 1 contains a table listing daily recession-discharge as read from the recession-curve and the corresponding volume of channel-storage. The reader is presumed to be acquainted with Horton’s papers relative to the discharge-channel-storage relationship [see 2 and 3].

Stream-regimen during recession is a function of the portion of the total flow derived from ground-water sources, as well as characteristics of channels. To this extent a relationship between total discharge and upstream channel-storage would vary with the relative amount of discharge derived from ground-water sources. The full expression for the recession-limb of a hydrograph is

\[
\text{Outflow} = \text{Ground-water inflow} + \text{loss in channel-storage}
\]

or

\[
\text{Outflow} - \text{ground-water inflow} = \text{Loss in channel-storage}
\]

The volume of channel-storage is more correctly related to total discharge than to the difference, outflow minus ground-water inflow. In this paper channel-storage volumes have been nevertheless related to outflow minus estimated ground-water outflow, the distinction between ground-water inflow and outflow being ignored. This relationship refers to the volume above the river-profile that would be maintained by ground-water discharge alone. However, this relationship is also affected by the height of the ground-water profile. Elementary consideration indicates that such effect would approach zero in a channel where the mean velocity does not sensibly vary with changes in stage. In those channels where the mean velocity increases with the stage an increased ground-water profile would decrease the volume of surface-water channel-storage at a given rate of discharge from such channel-storage. The reverse would be true where the mean velocity decreases with increase in stage.

However, it appears that the effect of ground-water upon the channel-storage relationship during flood is of minor consequence since ground-water flow is frequently an inconsequential portion of flood-discharge. At least, errors from this source do not approach those from other sources.

Figure 3 presents a comparative study of the computed discharge-channel-storage relations at the several stations in the White River Basin. This Figure shows the channel-storage in second-foot-days per square mile, against drainage-area at the several stations, when the rate of outflow is 1, 5, and 10 second-feet per square mile. As might be predicted the upward trend in unit channel-storage in a downstream direction is very marked. Channel-storage-relation graphs for stations on the same stream can be adjusted to conform with the mean trend and thus made consistent with one another and accidental errors eliminated. It may be observed that the trend-lines show appreciable divergence in a downstream direction, in such a way that at low flows (1 csm) the divergence shows less tendency to increase downstreamward than at high flows (10 csm). This observation confirms the deductions regarding comparative channel-capacities made in connection with discussion of the recession-curves. Moreover, this observation seems to demonstrate that the average slopes of the graph of discharge-channel-storage relation—the value of M in \(Q = K_d S^M [2]\)—are not the same at all stations on the same river-system, but are greater at upstream stations than at downstream stations. For example, the value of M for the West Fork of the White River at Muncie, Indiana, is about 1.2 and for the White River at Hazelton, Indiana, is about 0.7.

To compute the daily volumes of channel-storage during a flood-rise in the channel-system of a large basin, the daily storage is first computed at all most upstream gaging-stations by simple application of the curve of discharge-channel-storage relation to the mean daily net discharge, that is, observed discharge minus estimated base or ground-water flow. To the volumes
Second-foot days per square mile

**FIG. 3—COMPARISON OF DISCHARGE CHANNEL-STORAGE RELATIONS AT INDICATED GAGING STATIONS OF THE WHITE RIVER BASIN**

at the upper station are respectively added the volumes in the drainage-areas between the upper gaging-stations and the next lower gaging-station, computed as follows. Assuming that at two upstream gaging-stations A and B, the channel-storage corresponding to a certain rate of flow per square mile is represented by $A_s$ and $B_s$, respectively, and at the lower gaging-station, C, the channel-storage corresponding to the same rate of flow per square mile is equal to $C_s$, then the storage in the channels below gaging stations A and B and above C at a certain time may be approximated by multiplying the channel-storage, as determined by the discharge at the given time at the lower gaging-station C by the factor $(C_s - A_s - B_s)/C_s$. The same reasoning applies where there are any number of upstream gaging-stations, the factor in more general terms being $1 - \left(\text{Sum of channel storage at all upstream gaging-stations at same unit-rate of discharge divided by channel-storage at lower gaging-station at the same unit-rate of discharge}\right)$. The volume of channel-storage in the intervening area above station C, as thus computed, is added to the volume above the upper stations to obtain a value of the channel-storage above station C that is considered more correct than can be obtained by the use of the discharge-hydrograph of station C alone.

The computation is continued from station to station downstreamward adding increments in intervening storage, bringing in storage on tributary streams, and including daily volumes in artificial storage.

The above factor does not remain constant through all ranges of discharge because of the non-parallelism of the trend-lines shown on Figure 3, and previously discussed. However, by evaluating the factor at the mean unit-rate of discharge (second feet per square mile) prevailing during the given flood, a value will be obtained that may be used with confidence through the flood in question.

Although an approximation, this procedure affords a method of computing volumes of flood-water in channel-storage with respect to time and place. The part of the volumes of flood-water measured by the described method are illustrated on Figure 4. Since it omits that volume above the normal recession-profile extended upstream from the stage at each gaging station, the method inherently gives values of the channel-storage during rising stages that are low, although where a number of gaging stations may be used the error is probably not great. During recession after
the flood-peak has passed out of the basin there is probably little error.

Figure 5 shows the results of applying the daily changes in channel-storage during the flood of January 1937 in the Wabash River Basin above Mount Carmel, Illinois, (drainage-area 28,600 square miles) to the measured outflow at Mount Carmel. The graphs show the observed discharge resulting from a period of general rainfall over the basin between December 26 and January 26 and the adjusted hydrograph, which is assumed to be essentially equivalent to the rate of inflow into the channel-system. The hydrograph of observed discharge at Mount Carmel bears but a very general relation to the derived hydrograph of the rate of inflow. The several flood-rises of the hydrograph of channel-inflow may readily be related to rainfall. For purposes of comparison Figure 5 also shows a graph of mean daily precipitation over the Wabash River Basin.

To compute the average infiltration over the basin during the storm-period, the total storm-precipitation is divided into as many reasonably distinct storm-periods as may be recognized, and the associated runoff computed on the basis of the hydrograph of computed inflow into the channel-system. Unless sufficient time has elapsed between storm-periods to permit water in transit over ground-surfaces to reach the stream-channels, the difference between the rainfall and runoff must be corrected for such surface-detention to obtain the total infiltration during the respective periods. A plot of the difference between cumulative daily precipitation and the cumulative daily channel-inflow through the storm-period is of essential value in estimating initial detention, and determining infiltration during the storm. Several graphs showing this information together with related meteorologic and hydrologic data with respect to many small basins in the Ohio River Basin will be published by the Geological Survey in its forthcoming report [1] on the floods of Ohio and Mississippi rivers, January to February, 1937.

The infiltration is a function of the rainfall-supply and of the ground-surface conditions. When the total infiltration is known and sufficient knowledge of characteristics of rainfall are available, the limiting capacity of the soil to absorb water under the given conditions, termed the infiltration-capacity, may be computed. Many methods are available for this purpose. The following is intended to present a procedure of computation adapted to ready computation over wide areas, during prolonged storms, and where the usually limited data only are available.

Computation of the infiltration-capacity—Doctor Horton [4] has described a method of computing the average infiltration-capacity over a drainage-basin during widespread rain-storms where the runoff is known and the rates of rainfall are available as furnished by a recording rain-gage. The procedure described by Horton is directly related and largely conforms to the theory that surface-runoff occurs only when rain falls at a rate in excess of the capacity of the soil to absorb water under the conditions existing at the time or the infiltration-capacity. Aided by appropriate curves described by Horton, a value of the infiltration-capacity is found...
Some practical difficulties arise in an attempt to apply Horton's procedure to compute the infiltration-capacity over a number of drainage-basins, affected by the same widespread storm since any individual basin may be more or less removed from a recording rain-gage. Recording rain-gage records, today are still sparse and the selection of a base-gage to apply to a given basin is involved with uncertainties. In Horton's procedure greater emphasis was apparently placed on the characteristics of rainfall at the recording gages than on the geographical variation of these characteristics. Need has been recognized for a technique which would give more weight to the geographical variations in characteristics of rainfall and which would be capable of ready application and of producing satisfactory results.

A systematic method has been developed as described herein, which conforms to the theory of surface-runoff as defined above. The proposed method has been applied to the determination of the approximate infiltration-capacities in the Ohio River Basin during the storms of December 1936 to January 1937 [1]. The total infiltration as described in the previous section is equal to the difference between total storm-precipitation and the associated surface-runoff provided that changes in surface-storage have been accounted for and that evaporation-losses from non-infiltrated water may be neglected as inconsequential. After computing the total infiltration a suitable statistical criterion of duration in hours of rainfall has been derived so that division of the known total infiltration by the duration defined by this criterion, will yield the correct value of the infiltration-capacity and satisfy the equation that the total rainfall in excess of the infiltration-capacity = surface-runoff. Such duration will be termed herein the duration of significant rainfall and the corresponding rate, the significant rate of rainfall. It is apparent that the duration of significant rainfall lies somewhere between the total period of rainfall and the duration of rainfall-excess and its location in this range bears a relation to the infiltration-capacity. If the relationship of rate of rainfall and duration were a straight line, the duration of significant rainfall would be equal to the mean of the total storm-duration and the duration of rainfall at rates in excess of the infiltration-capacity. Also for a straight-line relationship the duration of significant rainfall would equal the duration of rainfall at a rate equal to one-half the infiltration-capacity. Either of these relations would afford a rough approximate method for determining duration of significant rainfall. But the relationship of rate and duration of rainfall is rarely a straight line, but rather is generally exponential in form. Consequently, some adjustments are necessary to arrive at a satisfactory approximation.

Six methods have been derived by empirical processes and the results compared. These methods fall into two categories, each related to the two approximate methods indicated above. Three methods are in the first category, A, based upon the combination of two durations, one in terms of the total rainfall-duration and the second in terms of the infiltration-capacity. Three are in the second category, B, based upon a single duration between the two durations just mentioned, but expressed in terms of the infiltration-capacity.

<table>
<thead>
<tr>
<th>Method</th>
<th>Formula for criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-1</td>
<td>( t = (t_T + t_f)/2 )</td>
</tr>
<tr>
<td>A-2</td>
<td>( t = (t_{.01} + t_f)/2 )</td>
</tr>
<tr>
<td>A-3</td>
<td>( t = 0.425t_{.01} + 0.58t_f )</td>
</tr>
<tr>
<td>B-1</td>
<td>( t = t_{.5f} ) Duration of rainfall at 0.5 of infiltration-capacity</td>
</tr>
<tr>
<td>B-2</td>
<td>( t = t_{.6f} ) Duration of rainfall at 0.6 of infiltration-capacity</td>
</tr>
<tr>
<td>B-3</td>
<td>( t = t_{.4f} + 0.007 ) Duration of rainfall at 0.4 of infiltration-capacity +0.007 inch per hour</td>
</tr>
</tbody>
</table>

In these formulas, \( t \) is the duration in hours of significant rainfall, \( t_T \) is duration of rainfall-intensities per hour equal to or in excess of "trace", \( t_{.01} \) is the duration of rainfall equal to or greater than 0.01 inch per hour, and \( t_f \) is the duration of rainfall equal to or greater than the infiltration-capacity.

The six methods listed above have been tested by trial within a range of 0.01 to 0.10 inch per hour in infiltration-capacity with 13 automatic rainfall-records. The results have been averaged and are compared on Table 1 which shows the relation between the values of the infiltration-capacity computed by the method indicated and the value of the infiltration-capacity com-
puted by the exact method. Methods A-3 and B-3 apparently give the best results.

<table>
<thead>
<tr>
<th>Method</th>
<th>Average ratio of computed value to correct value of infiltration-capacity per cent</th>
<th>Mean deviation of computed values from correct values of infiltration-capacity per cent</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-1</td>
<td>89</td>
<td>11</td>
</tr>
<tr>
<td>A-2</td>
<td>96</td>
<td>4</td>
</tr>
<tr>
<td>A-3</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td>B-1</td>
<td>98</td>
<td>7</td>
</tr>
<tr>
<td>B-2</td>
<td>105</td>
<td>9</td>
</tr>
<tr>
<td>B-3</td>
<td>100</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 1--Comparison of computed value of the infiltration-capacity with correct values

Inasmuch as these approximate methods are expressed in terms of the unknown infiltration-capacity, their application requires the use of successive approximations, although such approximations tend rapidly to converge upon the desired result.

The first step for computing the infiltration-capacity of a given basin is the preparation of a map of the region upon which are plotted at the points representing recording rain-gage sites, the durations of rainfall during the given storm, at rates equal to or in excess of one-half of the estimated average infiltration-capacity of the basin. At the same points there are likewise plotted the average total precipitation divided by the duration of rainfall as determined above, which quotient has been termed the precipitation-ratio. Two systems of smooth lines are now drawn upon this map, one being lines of equal duration of significant rainfall and the other being lines of equal ratios of total precipitation to duration of significant rainfall. These lines are so drawn that the product of the two factors at strategic points of intersection equals the known storm-precipitation, at such points. Figure 6 shows a map of the Ohio River Basin, showing a system of these lines for the storm of December 27, 1936, to January 5, 1937. It may be observed that the two systems of lines approximate a trajectory.

Two values of the duration of significant rainfall at any point can be determined from this map by reading off the duration directly as well as the average rate at the same point. The storm-precipitation at the given point, as determined from an isohyetal chart or other method is divided by the precipitation-ratio as read from the map as above and thus the second value of the duration is found. The average of the two may be used as the proper value of the duration of rainfall, equal to or in excess of the selected rate of precipitation. If the lines are properly drawn so that their product at all points is equal to the precipitation, the two values so derived will be equal. This suggested method for determining the duration of rainfall has an advantage in that it gives appropriate weight to varying precipitation over the basin as well as to any meteorologic features that one may wish to consider in drawing the lines of equal duration.

It may be observed on Figure 6 that the precipitation-ratio varied relatively less over the basin than the duration of significant rainfall. The former varied from a minimum of 0.073 inch per hour to a maximum of 0.295 inch per hour, whereas the latter varied between nine and 90 hours.

Precipitation at any point is equal to the product of the duration and the precipitation-ratio and at stations having different storm-precipitation, one or both of these factors must be different during the given storm. Figure 6 indicates that the rate-factor is subject to less variation than duration but that higher precipitation, nevertheless, is accompanied by a higher average precipitation-ratio as well as a higher duration. These observations have been confirmed by a separate study of total storm-precipitation, rate and duration experience at 43 recording rain-gage stations in the Ohio River Basin, during the storm of January 20 to 25, 1937. The average rainfall-experience during this storm may be summed up as follows: Stations with one inch of precipitation had an average precipitation-ratio of 0.10 inch per hour and a duration of significant rainfall intensity of 10 hours, whereas stations with 10 inches of precipitation had an average precipitation-ratio of about 0.15 inch per hour as well as a duration of 67 hours. In this example precipitation increased ten times, while the average precipitation-ratio increased 1.5 times and the duration 6.7 times.

As stated, Figure 6 related to the duration of rainfall at rates equal to or in excess of one-half of the estimated infiltration-capacity. Duration at other rates within a limited range may be computed in the following manner: The rate-duration relationship is generally of the exponential form, \( d = aK^r \) where \( a \) and \( K \) are constants and \( d \) is the duration of rainfall at rates equal to or in excess of rate \( r \). Therefore, if the duration \( d \) at a given intensity \( r \) is known, then the duration \( d_{r+1} \) at an intensity \((r+1)\) is \( K \times d_r \). Therefore \( K = \frac{d_{r+1}}{d_r} \). The value of \( K \) at a given recording rain-gage station during a storm is best expressed as the ratio of the dura-
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A map is then prepared upon which is plotted the value of $K$ at recording-gage records and lines of equal value of $K$ drawn upon it (see Fig. 7).

Sufficient data are now available for computing the approximate infiltration-capacity for any basin in the area studied provided the precipitation and direct runoff are known.
Example—For Greenville Creek Basin above Bradford, Ohio, we have:

(1) Total precipitation, December 27, 1936, to January 5, 1937 = 1.6 inches
(2) Total direct runoff = 0.45 inch
(3) Difference [(1) - (2)] = total infiltration = 1.15 inches
(4) Duration of precipitation at rates equal to or in excess of 0.08 inch per hour from Figure 5 is 16.5 hours
(5) Precipitation-ratio from Figure 5 is 0.082 inch per hour
(6) Second value of duration is 1.6/0.082 = 19.5 hours
(7) Average duration is (16.5 + 19.5)/2 = 18 hours
(8) Approximate value of infiltration-capacity f = 1.15/18 = 0.064 inch per hour
(9) Significant rate (method B-3) = 0.4f + 0.007 = 0.033 inch per hour
(10) Value of K from Figure 7 is 0.78
(11) Duration of precipitation at 0.04 inch per hour or greater is 18 x 0.78 = 14 hours
(12) Therefore, duration of precipitation at the significant rate of 0.033 is 16.8 hours
(13) Second and final value of infiltration-capacity is 1.15/16.8 = 0.069 inch per hour

References

L. L. HARROLD (1425 Van Buren Street, N.W., Washington, D. C.)—The paper presented by the author serves a very worthy purpose in that it again brings before the attention of the hydrologists and hydraulic engineers the importance of channel-storage in studies of rainfall and run-off.

The author is correct in the statement that the upper end of the recession-curve cannot exceed a limiting slope of 45°. This means that the difference in the discharge from day to day is decreasing and the hydrograph of stream-flow has a concave shape. In other words the recession-curve is that portion of the hydrograph which occurs after the point of inflection on the recession-side of the runoff-graph.

In determining the recession-curve only that portion of the recession-side of the runoff-graph should be used which is beyond the point of inflection. On Figure 1 of the author's paper many points indicate that they are to the left of the point of inflection due to the fact that the difference increases from day to day. The fact that differences in the daily flows as tabulated on Figure 1 increase until the flow has fallen to 25,000 cfs indicates that the average recession-graph from 60,000 second-feet down to 25,000 second-feet (Fig. 1) has a slope to the right of a limiting 45°-line and that the author has given weight to that portion of the recession-side of the runoff-graph which occurs prior to the point of inflection.

The author has presented a good method for arriving at a better figure for the channel-storage for large watersheds than that which would have been computed by using only one discharge-channel-storage relation. A word of caution, however, should have been made so that the figures of channel-storage in second-feet-days per square mile (Fig. 3) would not be used in the correction-factor. If these figures are used, it is possible to arrive at a negative storage-difference between stations on the stream-system. Storage in units of second-feet-days instead of second-feet-days per square mile should be used in the correction-factor in obtaining the difference in channel-storage between stations.

The writer computed the difference in storage between the West Fork of White River at Newberry, Indiana, and the East Fork of White River at Shoals, Indiana, and the White River at Hazleton, Indiana, using first a flow of ten cfs, and then one cfs, as a basis for computing the correction-factor. The difference in the total storage at White River at Hazleton as computed by the use of two different flows amounted to less than one per cent.

WALTER B. LANGBEIN—Mr. Harrold points out that in determining the normal recession-graph only that portion of the runoff-hydrograph which follows the point of inflection should be used. It is assumed that reference is made to the presentation in "Surface-runoff phenomena" by R. E. Horton [Pub. 101, Horton Hydrological Laboratory, February 1, 1935] for support of this statement. The theory as described by Horton is that inflow into channels from surface-runoff occurs previous to the time of the point of inflection, and that only after the point of inflection has been passed does the hydrograph represent drainage from channel-storage alone. The point of inflection in this connection is of hydrologic significance, marking the transition from the hydrograph of discharge as maintained by inflow into the channels which is concave downwards to that portion of the hydrograph representing simple channel-storage drainage and which Horton describes as always being concave upwards.

The significance of the point of inflection insofar as it may be indicative of the time of cessation of surface-water inflow is apparently lost when large basins are considered since the relationship between singular points of the distribution of rainfall and the resultant hydrograph of runoff are largely obscured. Consequently, the theory which attaches this significance to the point of inflection is applicable only to small basins. This restricted application is necessary for another important reason, namely, the geometrical form of the recession-limb representing drainage of channel-storage of large basins is not necessarily concave upwards as it almost always is for small basins.

Channel-characteristics vary widely between the range of low water and extreme flood-stages such as encountered in recent flood-experiences. These variable channel-characteristics
as they relate to conveyance and corresponding storage-volumes are necessarily reflected in the recession-limb.

Some discussion has already been given of this relationship. Although it may be impossible to write a single expression for the discharge and channel-storage volume for the entire range of stages experienced in a given river, it may be possible within a restricted range to write

\[ Q_c = KS_c \] and \[ Q_c = dS_c/dt \]

with notation as given in [2] and [3].

The exponent \( M \) is related to the conveyance and the physical dimensions of the tributary channel-system of the given basin. Values of \( M \) greater than unity seem to be characteristic of channel-systems in which all water moves downstream at efficient rates of discharge. Increasing volumes of impounded water either in lakes or over wide flood-plains reduce the mean velocity and result in lowered values of exponent \( M \). It is physically possible for all real positive values of \( M \) to be encountered. The value of \( M \) determines the geometrical nature of the recession-limb of the hydrograph. Five types or forms of recession hydrograph-limbs may be recognized in relation to the value of \( M \). These types and their geometrical characteristics are as follows:

**Type A: \( M > 1 \)**
1. Mean velocity, \( Q_c/S_c \), increases with rise in stage
2. Recession-curve concave upwards
3. Recession hydrograph-limb concave upwards

**Type B: \( M = 1 \)**
1. Mean velocity, \( Q_c/S_c \), is a constant
2. Recession-curve is a straight line through the origin
3. Recession hydrograph-limb, concave upwards, plots as straight line on semi-logarithmic paper

**Type C: \( 1 > M > 1/2 \)**
1. Mean velocity, \( Q_c/S_c \), decreases with rise in stage
2. Recession-curve concave downwards, as \( Q \to 0 \), the curve becomes asymptotic to the vertical, and as discharge increases the curve approaches a 45°-slope
3. Recession hydrograph-limb concave upwards

**Type D: \( M = 1/2 \)**
1. Product of mean velocity and discharge is a constant, \( S_c \times Q_c \) constant, mean velocity, \( Q_c/S_c \), decreases with increase in stage
2. Recession-curve is a straight line at 45°-slope but does not pass through origin.
3. Recession hydrograph-limb is a straight line.

**Type E: \( 0 < M < 1/2 \)**
1. Mean velocity, \( Q_c/S_c \), decreases with increase in stage
2. Recession-curve is concave upwards approaching a 45°-slope as discharge increases
3. Recession hydrograph-limb is concave downwards

It should be noted that the recession hydrograph-limb of Type E is concave downwards. This Type is not at all impossible. R. E. Horton [2] gives a table listing values of \( M \) which includes several medium-sized basins having recession-limbs approaching Type E.

The slopes of recession-curves of types A and B are steeper than 45° throughout their range, but it may be noted that the upper limits of recession-curves of types C, D, and E with increasing discharge each approach a limiting 45°-slope. This fact has served as the basis for the statement to which Harrold refers in the second paragraph of his discussion. Except for Type E the slope of the recession-curve is never less than 45°.

Very rarely does the recession-curve of a given basin fit the same one of these types throughout its range, although many small basins are of Type A and many approach Type B. The author has found but few recession-curves of Type E. However, it is desired to point out that such curves within a limited range are distinct possibilities and particularly so in the case of large rivers with wide overflow- or lake-areas. The upper end of the recession-curves of such rivers frequently are of Type E. The recession-curve of the Stillwater River below the Englewood Retarding Basin at Englewood, Ohio, is an example of this Type. The drainage-area above the gaging station is 646 square miles, but the fixed-outlet retarding-basin provides an artificial increase in storage so that the recession-hydrographs are of Type E.

When the level of the reservoir is below the top of the outlet conduits, the natural drain-
The age-characteristics of the drainage-basin are not modified and the recession-curve in the range of discharge below the outlet top is of Type A. The value of M in this range is approximately 1.2. However, when the level of the pool is above the top of the outlet conduits, the value of M is reduced to 0.35, making the recession-hydrographs of Type E and concave downwards. Hence, when the reservoir drains, the recession-limb of the hydrograph is concave downwards until the level of the top of the fixed-outlet conduits is reached, when the recession-hydrograph becomes concave upwards. This example is given in preference to a natural river-basin because the discharge-channel-storage relationship derived as previously explained in this paper and whose characteristics have just been discussed, was checked by addition of the known volumes of water in the reservoir at a given discharge at the gaging station and volumes of channel-storage above a gaging station on the Stillwater River above the Reservoir at Pleasant Hill, Ohio, (drainage-area 502 square miles). The recession-curve at the latter place is of Type A, the value of M being approximately 1.1.

The upper end of the recession-curve shown on Figure 1, to which Harrold refers, is of Type E. It is not considered essential to the purpose of this paper to support the accuracy of the particular curve shown on Figure 1; subsequent data may support it or not. However, its accuracy cannot be challenged without adequate study of the hydraulic characteristics of the tributary basin. The fact that the value of M for high discharge-values is less than 0.5 is not in itself sufficient evidence to discredit it. Mr. Harrold's remarks concerning Figure 3 are well taken. Volumes of channel-storage in a given problem should be reckoned in second-foot-days and not in terms of unit channel-storage as shown on Figure 3. This figure is presented to demonstrate variations in unit channel-storage and to suggest a method for adjustment of discharge-channel-storage graphs.

SYNTHETIC UNIT-GRAPHS

Franklin F. Snyder

Synopsis—This paper presents a method, mostly empirical, of deriving synthetic unit-graphs. It is of assistance in the study and analysis of runoff-characteristics of drainage-areas of from 10 to 10,000 square miles for which stream-flow records may or may not be available. No attempt has been made to eliminate the use or need of judgment and experience in such studies.

The "lag" or time from center of mass of rainfall to peak of runoff is the principal drainage-basin characteristic used in deriving the synthetic unit-graphs. An approximate method of determining the lag is given for use on areas with no stage-records available. The peak-rate of the unit-graph is expressed as a function of the "lag".

A distribution-graph is also determined by means of the lag. Knowing the peak-rate and the distribution-graph for the area in question, a unit-graph can then be constructed. An example of this procedure is given for the French Broad River Basin above Dandridge, Tennessee.

The additional use of the lag-characteristic in flood-forecasting is described and in conclusion, the limitations of the method are discussed.

Introduction—It is assumed that the reader is familiar with the basic theory and practical application of unit-graphs. If not, reference is made to them [see 1, 2, and 3 of "References" at end of paper]. As used by the author, a unit-graph is the discharge-graph of one inch of surface-runoff from a given area for a typical or specified type of storm of some unit of duration. The basic assumptions of the unit-graph method are that for such storms the time-duration of discharge of surface-runoff is a constant for any basin and the ordinates are proportional to the total amount of surface-runoff and distributed according to the unit-graph shape. A distribution-graph expresses the unit-graph by giving the discharge-volume of successive equal time-periods as a percentage of the total volume.

The procedure herein described for deriving unit-graphs was developed mainly for studying and forecasting stream-flow on areas with no immediately available records of runoff. However, it has been of material assistance in the study of areas for which discharge or gage-height records are available.

True unit-graphs are not directly obtainable from published mean daily discharges, although distribution-graphs based on 24-hour periods can be worked up and serve as an aid in outlining the unit-graph.