HYDRAULIC CRITERIA FOR SAND-WAVES

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Sand-waves on rivers are rhythmic successions of waves which occur at flood-stages of streams heavily loaded with sediments. They take their name from the fact that sand and associated silts and gravels form a large part of the load transported by a river at such times. They seem to be peculiar to the Southwest and many vivid descriptions of them can be found in the literature of that region. R. C. PIERCE [see 1 of "References" at end of paper], who observed many sand-waves on the San Juan River in Utah, has described them as resembling in appearance "the waves thrown up by a stern-wheel rivér steamboat." He further describes their appearances as follows: "The sand-waves are not continuous, but follow a rhythmic movement. At one moment the stream is running smoothly for a distance of perhaps several hundred yards. Then suddenly a number of waves, usually from six to ten, appear. They reach their full size in a few seconds, flow for perhaps two or three minutes, then suddenly disappear. Often, for perhaps half a minute before disappearing, the crests of the waves go through a combing movement, accompanied by a roaring sound. On first appearance it seems that the wave-forms occupy fixed positions, but by watching them closely it is seen that they move slowly upstream. In the narrow parts of the stream the waves may reach nearly the width of the river, but in the wider parts they occupy smaller proportional widths. Usually they are at right-angles to the axis of the stream, but at some places, particularly in the wider parts of the river, they may suddenly assume a diagonal position, moving rather rapidly across the stream in the direction toward which the upstream side of the wave has turned." Many such descriptions may be found which in the main bear out PIERCE'S account, varying, however, as to size of wave, rate, and sometimes as to direction of movement.

Sand-waves are significant to the extent that they are associated with sediment-transportation during floods when so much of the changes in channels take place. Other influences of sand-waves on height of flooding are probably more or less secondary.

The rise and fall of water in stream-channels in response to rainfall or snow-melt is called a flood-wave and in comparison the sand-waves are strong local undular waves superimposed on the long flood-wave, brought about by hydraulic factors of slope, depth, velocity, and load. It is not believed that changed rates of discharge brought about by fluctuations in rainfall is the primary cause, although PIERCE states that sand-waves on the San Juan appeared in their best development on rapidly rising stages, nor is it likely that their occurrence affects the progress of the flood-wave in any significant degree, although their occurrence is indicative of a channel-condition that is of particular influence on the movement of the flood-wave. They are fairly common, yet exact reasons for their formation are still moot questions. It is possible that an examination of the hydraulic factors associated with such waves as reported by competent observers may aid in advancing an ultimate solution. Unfortunately, with one or two partial exceptions, quantitative information is lacking; all that is available is a visual description of the waves, somewhat as reported by PIERCE.

GILBERT [2] in his now classic researches on the transportation of debris by running water reported that he observed three phases of the transportation of debris.

"When the conditions are such that the bed-load is small, the bed is molded into hills, called dunes, which travel downstream. Their mode of advance is like that of eolian dunes, the current eroding their upstream faces and depositing the eroded material on the downstream faces. With any progressive change of conditions tending to increase the load, the dunes eventually disappear and the debris-surface becomes smooth. The smooth phase is in turn succeeded by a second rhythmic phase, in which a system of hills travel upstream. These are called antidunes, and their movement is accomplished by erosion on the downstream face and deposition on the upstream face. Both rhythms of debris movement are initiated by rhythms of water movement" [2, p. 11].

"Usually each antidune occupied the full width of the experiment trough; and in natural streams, so far as I have observed, they either reach from side to side of the channel or else form well-defined rows in the direction of the current. Not only is a row of antidunes a rhythm in itself, but it goes through a rhythmic fluctuation in activity, either oscillating about a mean condition or else developing paroxysmally on a plane stream-bed and then slowly declining. Paroxysmal increase starts at the downstream end of a row and travels upstream, gaining in force for a time, and the climax is accompanied by a combing of wave crests. Where the debris is very coarse, as on the outwash plains of glaciers, a din of clashing bowlders is added to the roar of the water" [2, pp. 31-32].

These descriptions of the antidyne phase greatly resemble those of some types of sand-waves frequently observed in rivers. GILBERT'S experiments on load, slope, grade of sediment, depth, and discharges provide a means for determining the hydraulic conditions that are necessary for the formation of antidunes, with the possibility that the results may have application to studies of sand-waves in rivers.

Criteria for antidunes--GILBERT observed "the slopes at which the phases of traction change are lower for large streams than for small, and are lower for fine debris than for coarse."

Figure 1 shows for a single grain-size the phase of transportation in relation to the bed-slope and the product VR used as a measure of the "hydraulic size" of a stream. The latter product when divided by the kinematic viscosity is the REYNOLDS number and so becomes a measure of the viscous forces. The graph supports GILBERT by showing that lower bed-slopes for given changes in phase were associated with increase in the product VR.

However, slopes, as such, depend upon the friction-conditions peculiar to the experiment

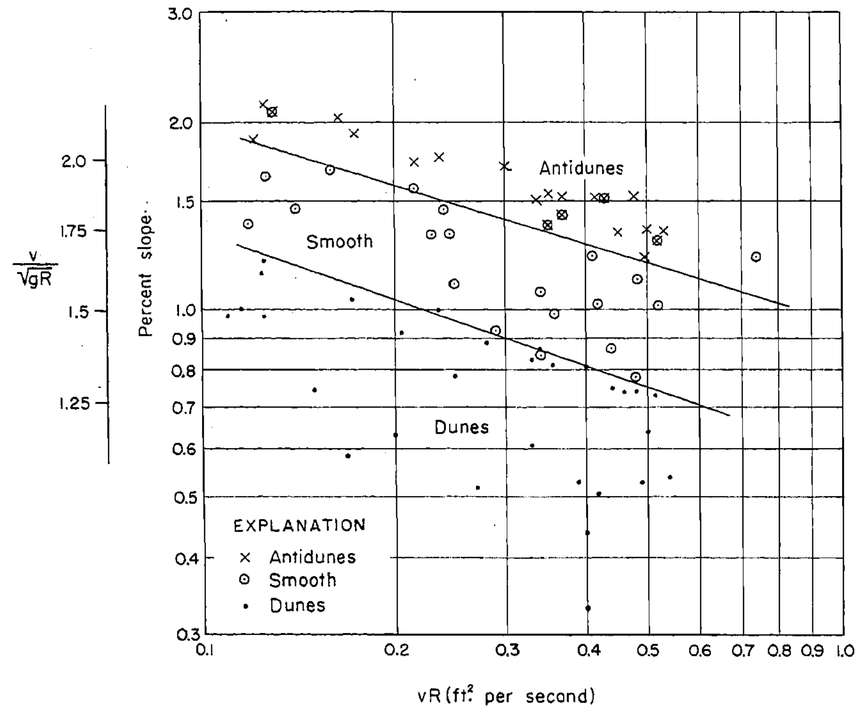

Fig. 1--Hydraulic criteria for phase of transportation of sediment, Gilbert's data, Grade C (0.50 mm)

trough GILBERT used.

To generalize the results on Figure 1, the slope should be multiplied by C^2 where C is the CHEZY friction-coefficient. But SC^2 according to the CHEZY equation is expressible in terms of velocity and depth which exist under the prevailing slope and friction. The ratio $(V/\sqrt{gR})$ so definable is dimensionless and is proportional to $\sqrt{SC^2/g}$. To define this proportionality a graph for each grade of sediment was plotted (not published) between the slope, S, for each run and the corresponding value of $(V/\sqrt{gR})$. The plotted points indicated a good relationship, which is shown as a secondary scale on Figure 1.

Figure 1 shows a scale along the vertical ordinate of $(V/\sqrt{gR})$ and the same conclusion made in regard to slope thereby applies in more generalized terms with reference to the kineticity $(V/\sqrt{gR})$.

Figure 1 applies to grain-size C of GILBERT'S experiments (0.5 mm mean diameter) but the same study has been made of four other sizes for which observations as to phase of transportation were made.

The results are shown in the tabular form below, giving the value of the kineticity, as the dimensionless ratio $(V/\sqrt{gR})$ is called, for change in phase of transportation. The position of the sloping lines as on Figure 1 for the several grades studied is indicated by the value of $(V/\sqrt{gR})$ corresponding to values of VR of 0.2 foot per second and 0.4 foot per second.

Table 1--Values of $(V/\sqrt{gR})$ at change in phase of transportation for indicated values of VR and grades of sediment

Dunes to smooth			Smooth to antidunes		
	VR = 0.2	VR = 0.4		VR = 0.2	VR = 0.4
Grade A (0.30 mm)	1.10	1.02	Grade A (0.30 mm)	1.33	1.20
B (0.37 mm)	1.31	1.18	B (0.37 mm)	1.67	1.40
C (0.50 mm)	1.46	1.36	C (0.50 mm)	1.76	1.69
D (0.78 mm)	1.45	1.35	D (0.78 mm)	a	a
E (1.72 mm)	1.50	1.39	E (1.72 mm)	a	a

[a]No observations reported by GILBERT.

The above grain-sizes are all within the coarse-sand range. A decrease in the value of $(V/\sqrt{gR})$ with decrease in grain-size is readily apparent, and suggests that with finer grades of sand or with silt or perhaps clay, even lower values of the kineticity but greater than 1.0 would be required to induce sand-waves.

The observations on the lower transition (dunes to smooth) are more complete. These results indicate an increase in kineticity from Grade A to Grade C, while Grades D and E were about the same as Grade C, averaging about 1.5. It is likely that the upper transition (smooth to antidunes) for coarser Grades D and E would be about the same as that for Grade C, 1.76 at VR = 0.2 and 1.69 at VR = 0.4.

The rate of decrease in the kineticity, $(V/\sqrt{gR})$ between VR = 0.2 and VR = 0.4 would indicate that for values of VR = 10 or 20 such as characterizes full-scale streams in flood that the kineticity would approach unity, and indeed sand-waves have been observed in rivers in which the ratio $(V/\sqrt{gR})$ was only slightly greater than 1.0. However, in comparing heavily sediment-laden streams, it seems necessary to consider their viscosity. This is done through use of the REYNOLDS'S number (VR/u) where VR is the product referred to above and u is the kinematic viscosity. Experiments reported by BINGHAM [3] indicate that the viscosity of a suspension of sediment in water increased with percentage of sediment, up to the point at which the fluidity of the mixture is reduced to zero and when it becomes a plastic solid.

Actual values of the viscosity of sediment-water suspensions are not available. However, descriptions by observers of sand-waves indicate that it must be noticeably greater than clear water. Viscosity at a given temperature probably is a function not only of per cent of solids in suspension, but probably of their texture, state of floculation or dispersion and therefore general values cannot be given. It is likely, however, that per cent of solids accounts for the greater part of the range in viscosities experienced. BINGHAM [3, pp. 291-292] summarizes: "For each temperature, the fluidity (reciprocal of viscosity) falls off rapidly and linearly with concentration of solid, so that at no very high concentration by volume the fluidity of zero would be reached. This concentration of zero-fluidity is independent of the temperature and is the concentration which serves to demarcate viscous from plastic flow."

BINGHAM also pointed out that at a given temperature and concentration finer textures produce higher viscosities.

The quantity of a sediment transported by a given stream at capacity varies with its velocity, depth, and gradient. Under the particular conditions prevailing in GILBERT'S experiment the percentage of sediment transported did not exceed about two per cent by weight. Sampling of rivers in which sand-waves occur indicate loads as high as 10 or even 25 per cent of suspended sediment. The kinematic viscosity under such conditions would be relatively high and the REYNOLDS number (VR/u) lower in proportion. Moreover in the scale of GILBERT'S experiments, the mode of transportation of the coarse sand was dominantly by bed-load traction. Reported observations of sand-waves indicate that the load seems to be in suspension, though large bed-loads of unknown amount are doubtless present.

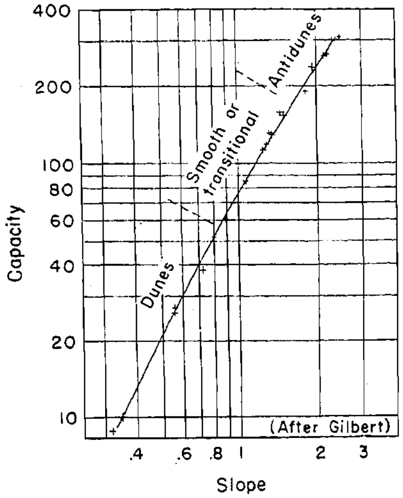

Fig. 2--Logarithmic plot of a series of observations on capacity and slope

Because of these differences and because of the unknown viscosities it is difficult if not impossible to apply the results of GILBERT'S experiments to full-scale rivers, except in the qualitative way described.

The load carried by GILBERT'S flume was at capacity and varied in accordance with hydraulic factors such as the velocity, depth, and gradient. There are many formulas available connecting these variables. However, it might be expected that some difference would be found in the relative efficiency for transportation of the phases of collective movement; the dune, smooth, and the antidune. If such existed a break in the relation between capacity and the related hydraulic elements would be expected; however, examination of GILBERT'S data as shown by Figure 2, for example, reveals no such difference in efficiency of transportation. For this reason studies of sand-waves may have little practical significance in fluvial morphology. However, sand-waves are indicative of torrential flow and supercritical slopes, which are important factors; for example, the same discharge could be carried in a channel of identical shape and material, with same total energy [= depth + $(V^2/2g)$] under which conditions it would have a lesser slope and lower velocity and its capacity and erosive ability reduced accordingly.

Although the average rate of transportation of debris over a long interval that may be seemingly unaffected by the occurrence of antidunes or sand-waves, it is likely, however, that local or short-period rates during the formation and breaking of the sand-waves differ greatly but to a yet unknown extent from the average. This variation from the average might nevertheless be important with respect to bed stability.

Morphologic conditions for sand-waves--The preceding discussion indicates that sand-waves occur only when flow is torrential, that is, when $(V/\sqrt{gR}) > 1$, which exists only when slope exceeds (g/C^2). Moreover, the effect of sediment-load (particularly the finer sizes) is to decrease the amount of kineticity required for the formation of sand-waves. According to STRICKLER, the CHEZY friction-coefficient C varies as $(R^{1/6}/d^{1/6})$ where R is the hydraulic radius and d is the median diameter of the bed material. Hence critical slope varies as $(d^{1/3}/R^{1/3})$ and is reduced by fines and increase hydraulic radius. Accordingly, sand-waves would be expected to be frequent flood-adjuncts under conditions of steep slopes and fine sediments, a combination that is characteristic of the southern intermountain region.

References

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[2] G. K. GILBERT, The transportation of debris by running water, U. S. Geol. Surv., Prof. Paper 86, 1914.
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MODIFICATION OF A THEORY ON THE RELATION OF SUSPENDED TO BED-MATERIAL IN RIVERS

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Using a statistical approach together with recently developed theory on turbulent flow, LANE and KALINSKE [see 1 and 2 of "References" at end of paper] arrived at a functional equation relating the amount of suspended material in a stream to the physical characteristics of the bed and channel. They obtain

$$(N_o/N_b) = \phi\ (t_c)$$