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THE PHYSICS OF THUNDERSTORMS

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Abstract--The physics of a thunderstorm is studied from the viewpoint of hydromechanics rather than thermodynamics, although the latter processes have not been entirely disregarded. The basic type of thunderstorm herein considered is the case in which the flow phenomena are similar to those existing in connection with two reservoirs with different pressures. The treatment of the various processes involved in a thunderstorm is mainly quantitative. Equations are developed based on the laws of continuity and fluid flow, from which the dimensions and characteristics of a thunderstorm can be determined.

Questions involving the limitation of rainfall splash, maximum limiting rain intensity of the thunderstorm, and the derivation of the characteristics of a thunderstorm, based on study of the rainfall pattern, are considered.

The various hydrological factors involved in a thunderstorm, such as condensation, evaporation, expansion, and lift are discussed.

Introduction and synopsis

A thunderstorm is much like a living organism; it moves, ingests moist air, and produces heat and energy to sustain it, and voids rain and drier air. It has the equivalent of organs which perform specific functions, much like that of the alimentary canal. It preserves its identity as a system while the substance of which it is composed is continually changing. It is born, grows to maximum stature, becomes old and feeble, and dies.

The expression thunderstorm rain is used in the sense of rain produced as a result of strong vertical or nearly vertical convection. The storm usually covers a small area, measured in tens rather than hundreds of sq mi, although, as a result of travel, it may produce rain over a much larger area.

Only one type of thunderstorm is here considered, in which the flow phenomena resemble those through a vertical or inclined pipe connecting two reservoirs with different pressures. This will be called a tubular thunderstorm. A more appropriate title for this paper would perhaps be: The Anatomy and Physiology of Tubular Thunderstorms and the Rains They Produce. The full meaning of this title will, however, be clear only when the paper has been read.

The subject has been treated more from the viewpoint of hydromechanics than thermodynamics, although the latter processes, where involved, are not disregarded. The paper is largely devoted to answering the question: What may be learned about the processes involved in connection with thunderstorms and their rains from meteorological observations taken at the ground level.

The independent variables here involved are barometric pressure, wind speed and direction, humidity, and rainfall, and their changes during the passage of the storm, and, from these observations, simple equations are developed from the law of continuity and the laws of fluid flow, from which the flow conditions in the dimensions of a thunderstorm and the characteristics of the rain which it produces can be determined. The treatment is quantitative and, as far as possible, rational, in that it is based on physical laws and processes.

The most important concepts involved are: (1) The kinematic model of a thunderstorm, subsequently described; (2) the concept of subdivision of moisture condensed in ascending air into two components: (a) The larger drops, which are precipitated on the area over which condensation takes place; (b) drops below a limiting size, dependent on the ascent velocity, which are carried aloft, spread radially in the outflow layer and are precipitated over an area considerably larger, usually several times larger, than the area over which condensation occurs.

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The space containing the ascending air current will be described as the convection tube, and the path of the center of its inlet end as it travels will be called the storm path. If the convection tube is inclined, the path of the center of the outlet of the tube, which governs the distribution of rain, will in general be parallel but not coincident with the storm path. Two principal cases occur: (1) Where the convection tube is vertical or so nearly vertical that its inclination has only an inappreciable effect on rain phenomena; (2) where the convection tube is so inclined to the vertical as to affect the rain phenomena, particularly rain intensity and distribution.

This study is directed toward development of methods for determining maximum station-rain intensity from meteorological data, and basic quantities used in the calculations are those for storms producing maximum or near maximum rain intensities. Fortunately for this purpose limiting values of data required, such as height of ascent, are more readily determined from known data than for other than maximum storms.

The author is indebted to Charles F. Brooks for supplying much needed data, and for two suggestions, made 25 years apart, that have played an important role in the development of the subject: (1) That thunderstorms travel with little regard to surface conditions but more as if they were carried along by an air current at some higher level; (2) That there is a difference between intense convective rains in Rocky Mountain regions and those occurring elsewhere in the United States, resulting partly from smaller increase of wind velocity with height in mountain regions, and partly from greater overheating of the ground surface in semiarid regions.

Characteristics of thunderstorms

The three thunderstorms illustrated by Figures 1 to 3 occurred in widely separated regions, yet the march of the meteorological elements during the passage of the storm over the observation station is very much the same. Another notable characteristic is the close synchronism of the principal changes in the different meteorological elements in the three cases.

It is evident that many well developed or major thunderstorms are similar in appearance and produce similar meteorological phenomena at the ground. The phenomena differ somewhat depending on the position of the observer relative to the center of the storm as it passes overhead. It seems that a typical thunderstorm represents a definite hydrodynamic system and has a definite rheologic or flow pattern, repeated over and over with variation in the elements, much as translatory ocean waves are repeated.

Kinematic model of a thunderstorm--In this study, the model method, often used in physics, has been followed. The kinematic model of a thunderstorm of the type here considered consists of a tube, vertical or inclined, gradually expanding upward to allow for expansion of air in ascent (see Fig. 4). The lower end of the tube is at mid-height of a surface layer of moist air overlain by a cover layer of at least potentially denser, stable air which is pierced by the tube. The tube discharges at a distance below the tropopause and thus constitutes a hydraulic system connecting two reservoirs with different pressures. The tube may be stationary or it may be carried along with the cover layer, while the outflow takes place into air flowing with the same or a different velocity and direction from the motion of the tube. Moist air entering the lower end of the tube has most of its moisture condensed above the condensation level, and in the hail and snow stages this condensation is converted into droplets of various sizes. The ascending air current separates these droplets into the following two components: (1) Large drops, having terminal velocities greater than the vertical component of ascent of air in the convection tube. These fall out as rain on the projected area of condensation. (2) Smaller droplets, which are carried aloft and discharged radially outward into the outflow layer. Drops below a certain size are evaporated and the radius of the outflow layer is such that if such a drop has ascended to the top of the layer and flows outward, falling at the same time with its terminal velocity, it will reach a limiting radius R in the same time required for it to fall through the cover layer. This fixes the maximum limit of the rain spread.

The rain phenomena which will occur in the operation of such a model can readily be determined for given conditions from the law of continuity and simple hydraulic formulas. The necessary data are obtainable from meteorologic observations commonly taken at the ground surface, including temperature, humidity, wind velocity and direction, barometric pressure, and rainfall. The results, computed for specific conditions, particularly as to rain intensity, area of rain splash, and rainfall distribution, can readily be compared with those actually occurring. A thunderstorm which produces results in accordance with the calculated results is designated a tubular thunderstorm. It is shown by comparison of observed and calculated data that most thunderstorms behave in the manner indicated by the model. In this manner the validity of the model is proven

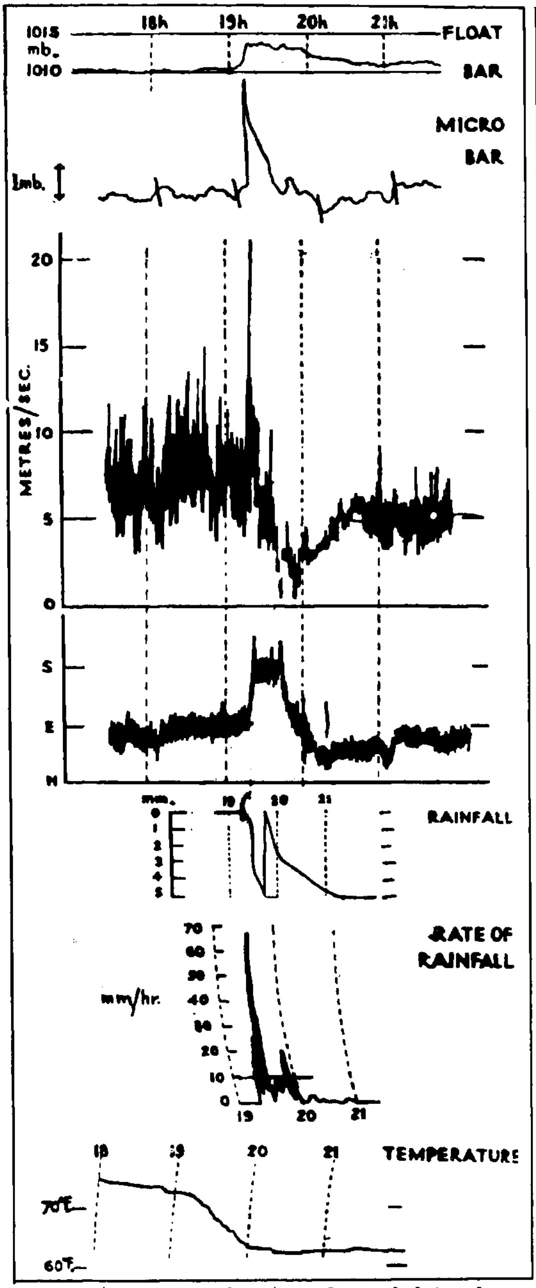

Fig. 1--Observed meteorological data of a thunderstorm

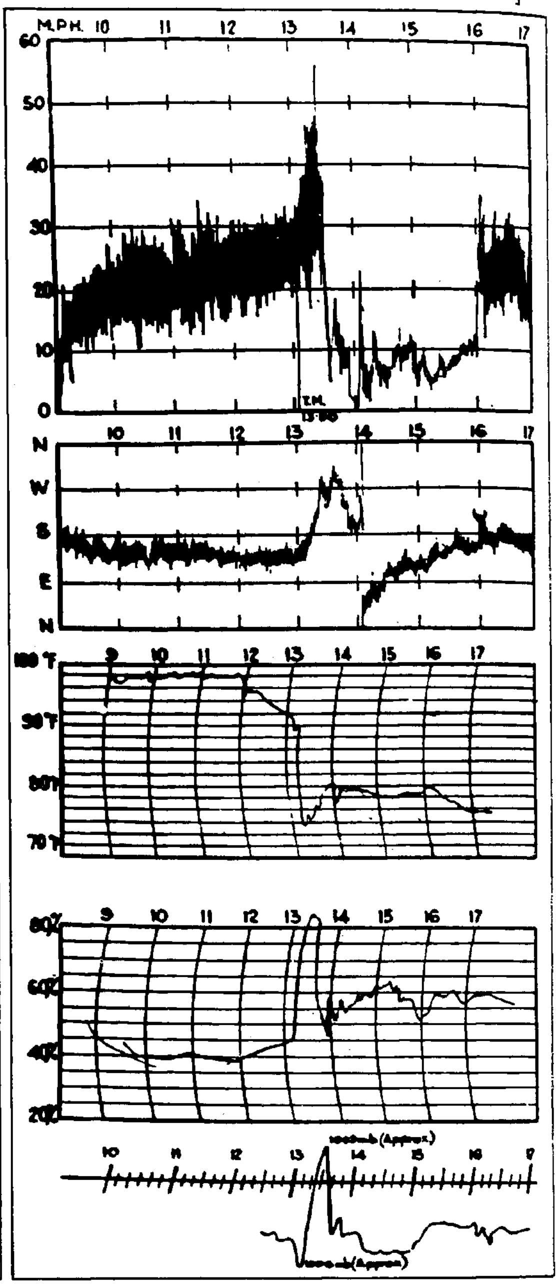

Fig. 2--Observed meteorological data of a thunderstorm

even though it may not accurately represent all the conditions involved. While a convective tube of circular cross section is assumed for purposes of simplicity, the utility of the model is not destroyed if, as is probably often the case, the convective upward current is not of circular cross section. This model reveals the thunderstorm mechanism and its operation stripped of the robe of cloud in which it is ordinarily enveloped.

The conditions favoring strong vertical convection are, as described by SHOWALTER [1944]: It is well known that pronounced rain activity is generally associated with air masses possessing a marked degree of convective instability which, in turn, is generally associated with a lower

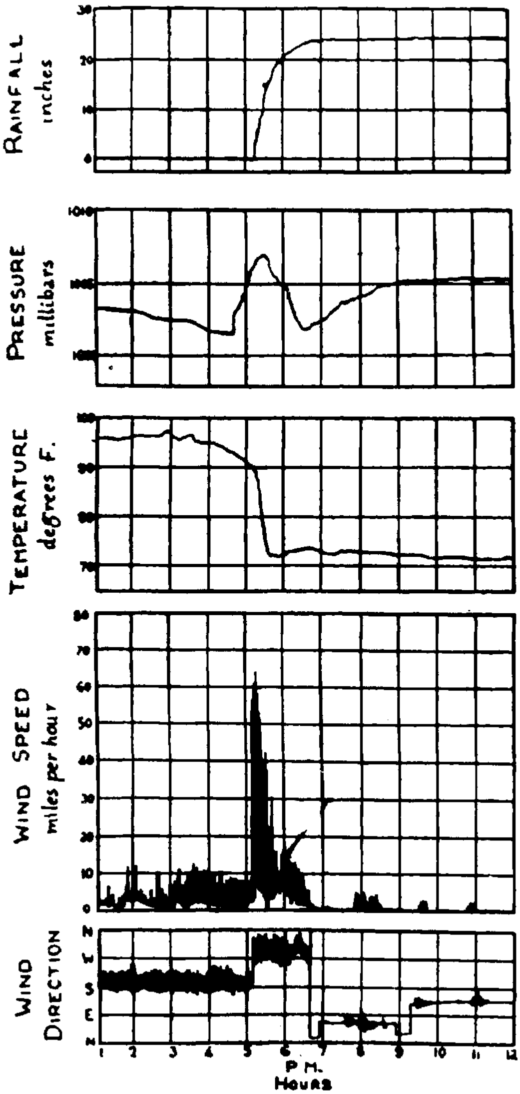

Fig. 3--Observed meteorological data of a thunderstorm

layer of unusually high moisture content separated from a much drier layer above by a stable layer or temperature inversion.

These conditions are provided by the model described. As shown in a separate paper [HORTON, in preparation], the ascending air current in most if not all vertically convective thunderstorms and hailstorms appears to be surrounded by a tubular sheet vortex. This is composed of a chain of ascending vortex rings of moist air forming a sheath around the convection core. Apparently this wall or sheath is thin or absent in intense thunderstorms and thick in hailstorms. It is not shown on Figure 4 since its presence or absence has little effect on the processes involved in thunderstorm rains as distinguished from hail.

The vortex rings composing the sheath are composed of moist surface air and preserve their identity as they ascend. Each ring is surrounded by field circulation which is composed of whatever fluid the vortex ring happens to be passing through at a given moment. This may be moist surface air from the convection core or dry country air from the surrounding region. Most commonly it is a mixture of both.

All conditions may occur from those where the vortex ring sheath is so thin as to be negligible, as in a typical convective thunderstorm with high ascent velocity, to those in which all the ascending moist air is entrained in the vortex rings surrounding the core. In the former case the core of ascending air is moist surface air which may become somewhat diluted with dry air brought in by the field circulation as it ascends. In the latter case the core is dry country air drawn in between successive vortex rings by the field circulation. The lower part of the envelope of cloud surrounding the vortex sheath is a mixture of moist air from the core and country air derived from the field circulation. At higher levels the cloud envelope is probably composed mostly of descending moist air from the outflow layer.

While not important in relation to rain production, certain other features of air circulation in thunderstorms throw light on thunderstorm structure. Acceleration of the downward-flowing air by falling rain at the border of the peripheral outflow ring tends to set up differential motion in the descending air current. At the point where this outflow turns nearly horizontal, at or above the cover layer, the differential velocity may produce violent turbulence or a small vortex. Such a vortex corresponds to the squall cloud often observed at the front of a thunderstorm. As noted by HUMPHREYS [1940], it rotates counterclockwise, the top of the vortex traveling toward the storm. This results from the fact that the downward velocity is greater in the peripheral rain belt near the storm core than at larger radii. The squall cloud lies outside the rain splash, consequently it cannot be a vortex formed by ejection of air at the break in the cover layer. The storm collar [HUMPHREYS, 1940] surrounds the base of the storm at a still greater radius. This is of the same nature but occurs at the outer limit of the down-flow and results from its motion relative to the country air.

That the descending air current surrounding the convection tube can be accelerated by rain falling through it is easily demonstrated. For a given drop size and suspension velocity v_s and with N drops per cubic meter, the falling drops induce an acceleration a' on the surrounding air.

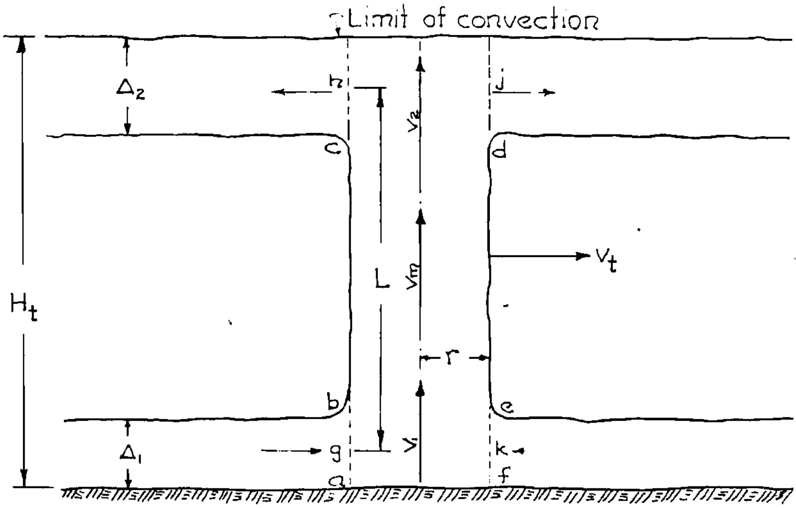

Fig. 4--Ideal convective storm

Since force = mass × acceleration, a′ = (Weight of drops per cubic meter)g/(Weight of air per cubic meter. For example, one two mm drop/dm^3 = 4.19 cm^3 or grams of water/m^3. Dry air at 0°C and 760 mm pressure weighs 1293.05 gr/m^3. For g = 981 cm/sec/sec the acceleration a′ is 3.15 cm/sec/sec. Rain in a descending air current can of course fall at a velocity relative to the Earth greater than its suspension velocity. If falling at a velocity of ten m/sec, the rain intensity would be roundly six in/hr, which is not an unusual rate for the short interval, usually less than 20 min, required for the front peripheral rain to pass over a given point.

Many sketches have been published purporting to represent a cross section of a convective thunderstorm [BLAIR, 1942; ALBRIGHT, 1939; WILLETT, 1944; and BRANCATO, 1942]. These sketches are guardedly indefinite as to the air currents in the interior of the storm, which is concealed by cloud. All such sketches which the author has seen either show or inferentially require the existence of a downward return current in the center of the ascending air stream. The type of flow required to support this theory is unknown in hydromechanics. (The central downward flow is characteristic where the falling rain is heaviest, and is a combination of rain drag and thermal currents.--C. F. Brooks). Apparently it could not exist in the presence of the free opportunity for a return current in the region surrounding the ascending air column with much reduced resistance to flow. It seems probable that the authors of such sketches have been over-influenced by certain physical phenomena, such as the squall cloud, the storm collar, and the apparent outrush of air at the onset of the storm. These phenomena are equally well or better explained by the thunderstorm model of the convective tube type. Probably the explanation of the outrush of air is not as simple as it appears superficially and may be in part the result of field circulation of the vortex ring forming the squall cloud.

In the sketches referred to, the cloud and rain patterns are those of a vertically convective thunderstorm, while the air circulation shown closely resembles that which would occur in a solitary or rolling wave. Probably this circulation does occur in the roll cloud which often accompanies the passage of a cold front. A vertically convective thunderstorm does not produce a long narrow rain splash at right angles to the direction of its travel. Instead it produces a more or less circular rain splash.

Fulks' diagram--The thickness Δ_1 = ab (see Fig. 4) of the inflowing layer of moist air is related to surface dew-point temperature θ_d. A curve showing this relation, as determined by the Hydrometeorological Section, United States Weather Bureau [ANONYMOUS, 1941] is shown on Figure 5.

For purposes of calculation the problem may be treated as if the convective tube extended from the level $\Delta_1/2$ up to h_1 at the center of the outflow layer, and inflow and outflow all occurred at these levels. It is necessary to determine the dew-point temperature at these levels. This and other elements of the computation are most readily carried out by the use of a thermodynamic diagram of the atmosphere. The most complete diagram is that of NEUHOFF [1910]. A simplified diagram which serves present purposes is that of Fulks (see Fig. 6). On this diagram vertical

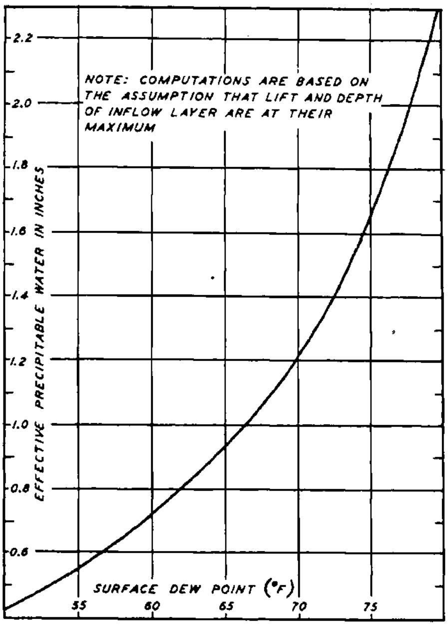

Fig. 5--Relation between temperature and depth of layer of moist air

lines represent temperature, °C; horizontal lines represent pressure, in mb; lines sloping slightly downward from left to right show elevations in km corresponding to different pressures and temperatures; curved lines sloping upward to the left are pseudoadiabats showing temperature and pressure at a given altitude in a mass of rising air saturated at the surface and cooling as it rises at the pseudoadiabatic lapse rate, that is, the rate at which temperature decreases in an ascending mass of saturated air cooled by adiabatic expansion. Curved lines sloping upwardo the right represent rates of condensation in mm/hr in such an ascending air mass, with a vertical depth of 100 m. For example, a mass of saturated air at 20°C, starting at the surface, would follow a course parallel with the adjacent pseudoadiabats and would attain a temperature of 16°C and a pressure of 900 mb at one km elevation, a pressure of 800 mb and a temperature of 11°C at two km elevation, and so on. When the center of this air mass is at two km elevation, it would produce condensation at the rate of approximately 0.7 mm/hr for each m/sec of ascent velocity.

Lift--Maximum condensation rate for a given ascent velocity will occur in conjunction with maximum dew-point temperature and maximum height of ascent of the air column. For purposes of calculation the height of ascent or lift will be taken as the vertical distance from the center of the inflow layer at level gk, Figure 4, to the center of the outflow layer at level hj. Because of expansion in ascent of the air the thickness of the outflow layer Δ_2 is greater than the thickness of the inflow layer Δ_1. (It is assumed here that the speed of outflow at the top is less in proportion to that of inflow at the bottom than the ratio of the densities. It must be borne in mind, however, that with a certain pressure gradient the flow increases in proportion to the decrease in density, which would tend to keep Δ_2 from getting much greater than Δ_1.-- C. F. Brooks) The mass of outflow is the same as the mass of inflow but because of expansion in ascent the diameter and velocity at the top of the convection tube and the thickness of the outflow layer will all, in general, be greater than at the inflow level. In more intense storms the outflowing air contains so little moisture that considerable variation in the assumed height of lift makes relatively little difference in the calculated condensation rate. In intense thunderstorms the tops of cumulo-nimbus clouds often extend nearly to the stratosphere. The extreme top of the cloud contains so little moisture that it may be neglected and maximum condensation may be taken as that occurring with maximum surface dew-point temperature and with the top of the outflow layer at the tropopause, or base of the stratosphere. Table 1 gives values that correspond to conditions in northern and eastern United States [ANONYMOUS, 1941].

Table 1--Cloud heights, tropopause, and freezing level

Item	Summer		Winter	
	ft	km	ft	km
Cumulo nimbus cloud tops				
Maximum elevation	52,000	16	30,000	9.0
Average elevation	16,000	5	11,500	3.5
Average base of stratosphere	43,000	13	33,060	10.0
Average freezing level	20,000	6	10,000	3.0

To simplify computations for maximum conditions the assumption may be made, as has been done by the Hydrometeorological Section of the United States Weather Bureau [ANONYMOUS, 1941] that for a maximum condensation the top of the outflow layer is at the level h_t of the base of the stratosphere and that the height of lift, $L = 2h_t/3$. Then since

$$h_1 = \text{mean inflow level} = \Delta_1/2 \quad (1)$$

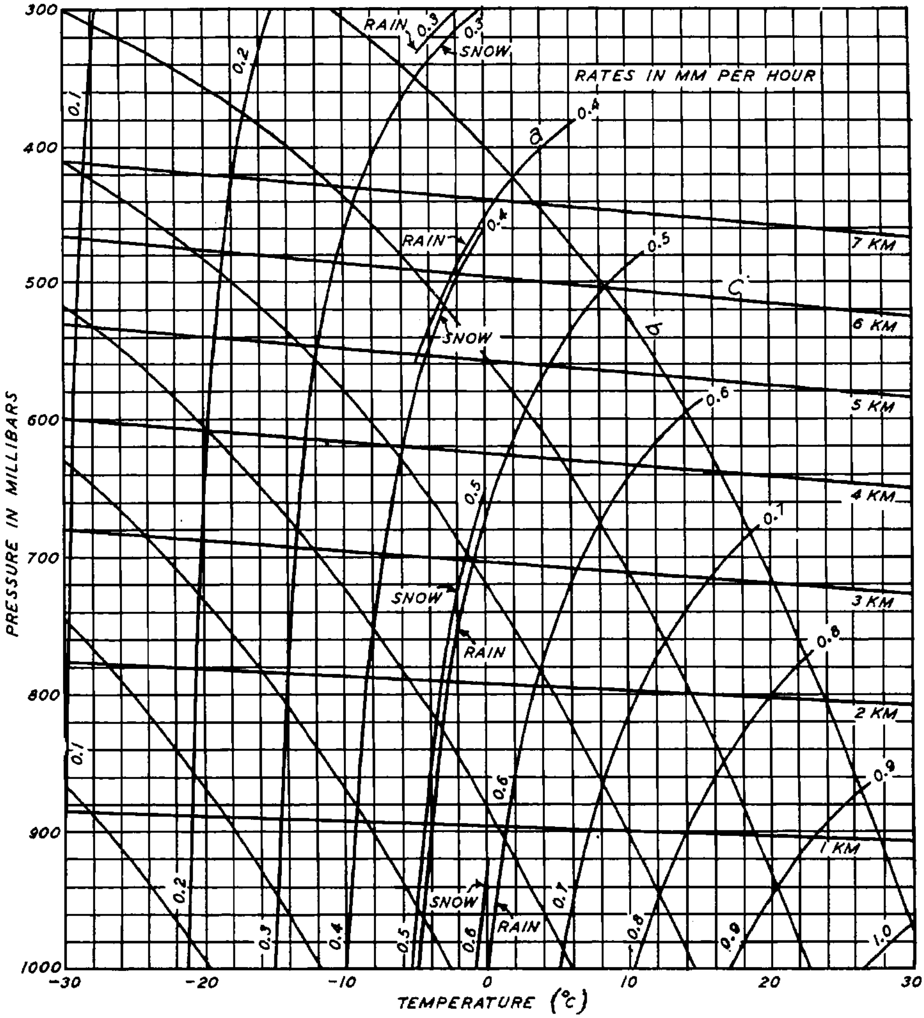

Fig. 6--Fulks' diagram for adiabatically ascending air [Monthly Weather Review, 1935]; a = rate of precipitation, rain or snow, for a 100-m layer with a vertical velocity of one m/sec; b = saturated adiabats; and c = altitude lines

it follows that

$$h_2 = \text{mean outflow level} = \Delta_1/2 + 2h_t/3 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

The thickness of the outflow layer is

$$\Delta_2 = 2h_t/3 - \Delta_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

Condensation level--For the purpose of determining maximum thunderstorm rain it is assumed that the air is saturated from the ground surface upward to the base of the stratosphere and is at its extreme or limiting maximum dew-point temperature at the ground surface. Ordinarily the surface air just preceding the storm has a temperature somewhat above the maximum dew-point temperature and the condensation level is above the ground surface. The lapse rate is then adiabatic up to the condensation level (1°C per 100 m) and the height of the condensation level in km is

$$h_c = (\theta_0 - \theta_d)/10 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Expansion--A mass of air of diameter $2r_1$ and of unit height at the inlet level, will have a volume $V_1 = \pi r_1^2$. If the expansion ratio is ρ_e, then at outflow level the volume is

$$V_2 = \rho_e V_1$$

or

$$V_2/V_1 = \rho_e \qquad (5)$$

If expansion takes place equally in all directions the air mass at the upper level will have a radius r_2 such that

$$\left.\begin{aligned} 2r_2 &= 2r_1 \rho_e^{1/3} \\ r_2/r_1 &= \rho_e^{1/3} \end{aligned}\right\} \qquad (6)$$

and a height $\rho_e^{1/3}$, so that its volume will be

$$\pi r_1^2 \rho_e$$

The imaginary walls of the convection tube may be considered as extended downward to the surface and upward to the tropopause, as shown by dotted lines on Figure 4. From (6)

$$r_2/r_1 = \rho_e^{1/3}$$

If

$$\Delta_2/\Delta_1 = \rho_e^{1/3}$$

then

$$v_0/v_h = r_1\Delta_1/r_2\Delta_2 = 1.0 \qquad (7)$$

The last result shows that if the thickness of the outflow layer is proportional to the vertical component of expansion, then the horizontal velocity of outflow across the walls of the convection tube will be the same as the horizontal velocity of inflow, both taken across the walls of the convection tube extended, and both relative to its rate of travel, not relative to the Earth. Under these conditions the horizontal outflow velocity is not affected by expansion, in other words, the outflow area is increased in the same ratio ρ_e as the volume and consequently the outflow velocity, $v_0 = v_h$. Also

$$V_1 = \pi r_1^2 v_1$$

$$V_2 = \pi r_2^2 v_2 = \rho_e V_1$$

and since

$$(r_2/r_1)^2 = \rho_e^{2/3}$$

$$v_2/v_1 = \rho_e^{1/3} \qquad (8)$$

Equation (8) shows that the average ascent velocity at the outflow end of the convection tube is greater than at the inflow end in the ratio $\rho_e^{1/3}$. The maximum ascent velocity, other things equal, will occur somewhat below the center of the outflow layer because the ascending air begins to spread at the base of the outflow layer. For practical purposes it is assumed, unless otherwise noted, that the vertical velocity at the center of the outflow layer is sensibly the same as the maximum ascent velocity. The mean ascent velocity will be the integrated average velocity from the inflow to the outflow level. Approximately

$$v_m = (v_1 + v_2)/2 = (1 + \rho_e^{1/3})v_1/2 \qquad (9)$$

For ρ_e = 2.0 this gives

$$v_m = 1.13 \ v_1$$

$$v_1 = 0.885 \, v_m$$

$$v_2 = 1.12 \ v_m$$

$$= 1.27 \ v_1$$

From the gas laws, $PV = R_g T_K$, where T_K = absolute temperature, Kelvin; R_g = the gas constant for the system of units used.

Using subscript 0 for ground level or standard conditions,

$$PV/P_0V_0 = T/T_0 \quad \ldots\ldots (10)$$

and the expansion ratio is

$$\rho_e = (P_0/P)(T/T_0) = (P_0/T_0)(T/P) = R_gT/P \quad \ldots\ldots (11)$$

where

$$R_g = P_0/T_0 \quad \ldots\ldots (12)$$

Using P_0 = 29.9 inches = 759 mm = 1013 mb, and T_0 = 86°F = 30°C = 303°K, which is about maximum dew-point temperature, R_g = 0.09868 or roundly 0.1 for P in inches and temperatures Kelvin.

Using seven km as approximately the base of the outflow layer for maximum storms and taking temperatures and pressures at this level from Fulks' diagram, for ground temperatures as shown in column (1) of Table 2, values of the expansion ratio ρ_e are obtained as shown in column (8).

Table 2--Expansion ratios

θ_d		θ_d		T/T_0	P_{mb}	P_0/P	ρ_e
C	Kelvin	C	Kelvin				
1	2	3	4	5	6	7	8
15	288	-26	247	0.856	412	2.43	2.08
20	293	-16	257	0.877	421	2.38	2.08
25	298	- 6	267	0.896	432	2.32	2.08
30	303	+ 4	277	0.914	442	2.26	2.08

For the assumed conditions the expansion ratio ρ_e comes out so close to the constant value 2.00 that ρ_e = 2.00 may be used without sensible error for conditions usually prevailing in maximum convective storms. It is true that the summits of cumulus clouds are sometimes carried above the tropopause and to some distance into the stratosphere. This apparently results chiefly from their momentum. These extreme summit clouds, however, produce little if any rain above the base of the stratosphere, and the conditions assumed suffice for present purposes.

Condensation rate--Before taking up flow phenomena in the outflow layer it is necessary to give some consideration to condensation rate and its relation to rate of rain production. The terms volumetric condensation, C_r, and volumetric rain intensity, V_p, will be used to describe, respectively, the total amount of condensation per unit of time in the convection tube and the total volume of rain precipitated on the rain splash area per unit of time.

Condensation rate is a function of two factors: Vapor content of ascending air, and rate of ascent. The volume of air entering the base of the convective cylinder bcdo (see Fig. 4) is $V_1 = \pi r_1^2 v_1$. At maximum height, $v_m = \pi r_m^2 v_m$. At outflow, $V_2 = \pi r_2^2 v_2$. The differences of volume resulting from expansion are reflected in the values of r and v.

Each unit of volume contains a mass of vapor m1 equivalent to an equal volume of liquid water in cubic centimeters. The air is cooled as it rises and expands, but the mass of air leaving the top cd of the column, exclusive of water vapor, is the same as that entering the base. Because of the lower temperature resulting from expansion, the vapor content m_1 per unit of mass or per unit of original volume, is reduced in ascent to m_2.

If $m_c = m_1 - m_2$ the mass in grams or the volume in cc of vapor condensed out of one cubic meter of air at this temperature and pressure at entrance to the convection tube will be

$$C_r = \pi r_1^2 v_1 m_c \quad \ldots\ldots (13)$$

For $\rho_e = 2.00$, $r_m = 1.13 r_1$ and $v_m = 1.13 v_1$. This gives

$$\left.\begin{aligned} C_r &= (1/1.13)^3 \pi r_m^2 v_m m_c \\ &= 0.693 \quad \pi r_m^2 v_m m_c \end{aligned}\right\} \quad \ldots\ldots (14)$$

If m_c is in gr/m^3 and r and v in m and m/sec, then C_r will be in cm^3/sec.

Expressed in depth on the projected mean cross-section area of the convection tube

$$\left.\begin{aligned} c_r &= 0.693 \ v_m m_c/1000 \text{ mm/sec} \\ &= 0.10 \quad v_m m_c \text{ in/hr} \end{aligned}\right\} \quad \ldots\ldots (15)$$

This is 1/10 in/hr of condensation per gr per m^3 of condensible vapor in the original area, and per m per sec of mean velocity of ascent.

If there were no intermediate loss of water, then the volumetric precipitation rate would be $V_p = C_r$. There are, however, intermediate losses.

Storage and dissipation of vapor by clouds--If a given volume of water is condensed from an ascending air current, then the volume available to produce rain is reduced by the volume which is entrained and later dissipated as water droplets in cloud. It is commonly assumed that this is negligibly small. However, under conditions existing in a violent thunderstorm this may not be the case.

Let V_1 = volume of water in cloud droplets, cc/m^3. The equivalent depth in mm is $1000\ hV_1/10{,}000$ for h in km.

Using WEGENER'S [1940] measurements of water content of clouds in the Swiss Alps, V_1 = 0.35 to 4.8 gr/m^3. For an average value of $V_1 = 2.50\ gr/m^3$, d_2 = 0.25 cc/km = 2.5 mm/km. For a cloud column 10 km in height this is equivalent to 25 mm or one inch depth of water. Results of the same order are obtained using Humphreys figure of 120 cloud droplets, 33 microns in diameter per cm^3 of dense cloud [HUMPHREY, 1940]. This gives 21.6 mm or 0.84 inch of water equivalent in a 10-km cloud column.

The cloud mass in a convective cumulus cloud may be considered as a cylinder ten km in height and ten km base diameter. Such a cloud mass would contain roundly 785 km^3, and with 2.5 cc cloud droplets per m^3 the total volume of water contained in cloud droplets would be 1962×10^9 cc total ($1\ m^3 = 10^6$ cc). This is equivalent to 1,962,000 m^3. This enormous quantity of water may become inactive as cloud. In the example given this water would be distributed over an area 78.5 km^2 and would be equivalent to a depth of 2.5 cm.

During early stages of a convective storm, the accumulation of moisture storage as cloud continues at such a rate that condensation rate minus drop formation rate equals rate of accretion to cloud storage. Meanwhile evaporation takes place from the increasingly larger exposed cloud surface. Equilibrium is attained when, for a steady storm, rate of accretion to cloud equals evaporation loss from cloud surface. Previous to this time, that is, in the early stages of the storm, the volumetric precipitation may be and usually is much less than the volumetric condensation rate. After the steady state is attained, a constant volume of cloud storage is carried forward with the storm and finally dissipated when the storm subsides. This volume of cloud is in effect abstracted from condensation in the early stages of the storm and, after equilibrium is attained, cloud storage has no further direct effect on the precipitation rate.

Evaporation--After cloud storage becomes filled, precipitation takes place, barring minor variations from gustiness, at a rate equal to the condensation rate, excepting for loss by evaporation.

The rate of evaporation from a cloud surface at high altitude is unknown and involves several factors: Low temperature, effect of wind, and rate of evaporation from drops as compared with that from a plane surface. Evaporation from a standard Class A, United States Weather Bureau pan rarely exceeds 0.02 in/hr as a maximum. Evaporation from small droplets may take place at much higher rates per unit of surface, as has been shown theoretically and experimentally. The evaporation from cloud surface may conceivably equal and perhaps exceed that from a plane surface in a similar environment, but even then, because of the extremely low temperatures, it is doubtful if it is ever as much as 0.01 in/hr depth on the projected rain splash area.

The total surface exposed to evaporation equals (number of droplets per unit of surface) × (surface per drop) × (total cloud surface).

Evaporation occurs not only from the surface layer of cloud droplets but to a small depth within the cloud. For 2.5 gr of cloud droplets per m^3 and 100 droplets per cm^3, with droplets 0.338 mm in diameter, the drop surface is 0.0359 m^2 per m^3 of cloud. If evaporation extends to one m depth and at a rate of one mm per hour, or roundly one inch depth per day, the evaporation would be 0.036 mm per hour on the exposed surface. For a cloud evelope equivalent to a cylinder ten km in diameter and tem km in height, this would be equivalent to 0.144 mm depth per hour on the basal area of the cylinder or roundly 0.1 inch per day. This water is derived from the ascending core of air through the field circulation and is replaced as fast as it is lost. Since it requires usually less than one hour for a rain splash to pass a given point, this loss is relatively small compared with the rain splash intensities commonly occurring in intense convective storms.

Rheology of thunderstorms--The term "rheology" connotes something more than motion or kinematics. It also includes the physical nature of flow processes. While this term has hitherto generally been used chiefly in connection with flow of plastic substances, the extension of its use to the condition here under consideration seems appropriate.

The term country wind, analogous to country rock of the geologist, will be used to describe the prevailing surface wind at a sufficient distance from the storm center to be unaffected by the storm.

Using the thunderstorm model, Figure 4, it will be assumed that the horizontal velocity v_h of flow across the walls of the convection tube at its base is constant all around its circumference. This is the velocity relative to that of the convection tube, not that relative to the Earth.

If there is no country wind at ground level the storm is carried along with the break in the cover layer at a velocity v_t. If the absolute values regardless of directions of the wind velocities at the front and back of the storm at ground level relative to the Earth are $|w_f|$ and $|w_b|$, then for the same velocity v_h of entry around the convection tube,

$$\left.\begin{aligned} |w_f| &= v_t + v_h \\ |w_b| &= v_t - v_h \end{aligned}\right\} \qquad (16)$$

Inflow to the convection tube will then continue uniformly around its circumference. Thus there is a strong ground wind at the front directed toward the storm and a weaker ground wind at the back from the opposite direction. If $|w_f|$ and $|w_b|$ are known

$$v_t = |w_f| - v_h = |w_b| + v_h \qquad (17)$$

From Figure 4

$$\pi r_1^2 v_1 = 2\pi r_1 \Delta_1 v_h \qquad (18)$$

and

$$v_h = r_1 v_1 / 2\Delta_1 \qquad (19)$$

Also

$$\Delta_1 = r_1 v_1 / 2 v_h \qquad (20)$$

The time $T = t_2 - t_1$ between maximum wind velocities at the onset and end of the storm with respect to an observer in the path of the storm center, is the time required for the storm to travel a distance $2r_1$ or

$$v_t T = 2r_1$$

and

$$\left.\begin{aligned} v_t &= 2r_1/T \\ r_1 &= v_t T/2 \end{aligned}\right\} \qquad (21)$$

From (17) and (19)

$$v_h = (w_f - w_b)/2 = r_1 v_1 / 2\Delta_1 \qquad (22)$$

or

$$\Delta_1 = r_1 v_1 / (w_f - w_b) \qquad (23)$$

Δ_1 may also be determined for given conditions from Figure 5 or, if available, from upper air observations.

At the center of the convection tube, v_h becomes zero and if there is no country wind, the measured wind velocity w_c at this point equals v_t, the translational velocity of the convection tube. This value v_t may be used to determine r_1 and Δ_1. This gives

$$r_1 = w_c T/2 \qquad (24)$$

As shown later there is usually a maximum of smoothed rain intensity with respect to an observer in the path of the axis of the storm at the time the front of the convection tube reaches him. There is a secondary maximum when the back of the tube passes the observer. The difference of these times may be used as the value of T in (21) to determine r_1. Another method is given later. When data permit two or more of these methods to be applied, the average determination or more consistent determinations should be used.

The methods described for determination of r_1 and Δ_1 become unreliable if the ground level observations are not taken in or close to the path of the center of the storm or if there is a country wind. In the latter case the component of the country wind as determined somewhat remote from the storm in the direction of storm travel must be added algebraically to w_c.

Ascent velocity, first method--The preceding analysis provides a means for determining the ascent velocities v_1 and v_2 if, as is assumed, the thickness Δ_1 of the inflow layer is known.

From (19)

$$v_1 = 2\Delta_1 v_h / r_1 \qquad (25)$$

Assuming that an approximate value of Δ_1 has been obtained, and that w_f, w_b, T, and w_c are known, v_1 can be determined from the equations

$$v_h = (w_f - w_b)/2$$

$$r_1 = w_c T/2 = v_t T/2$$

and

$$v_1 = v_2 = 2\Delta_1 v_h / r_1$$

Ascent velocity, second method--The maximum ascent velocity v_g may be calculated from the height J of the familiar barometric jump, which occurs just at the onset of the storm. BUELL [1943] has given the approximate formula

$$v_g = \sqrt{2R_g\,(T_s/P_s)J} \quad \ldots\ldots \quad (26)$$

where R_g = gas constant, T_s and P_s = surface temperature, Kelvin, and surface pressure, mb; J = height of barometric jump, mb. For T_s = 300°K (80.6°C) and P_s = 1000 mb, this equation can be reduced to the form

$$\left.\begin{aligned} v_g &= \text{(in ft/min)} = 2580\sqrt{J} \\ v_g &= \text{(in ft/ sec)} = 43\sqrt{J} \\ v_g &= \text{(in m/ sec)} = 13.1\sqrt{J} \end{aligned}\right\} \quad \ldots\ldots \quad (27)$$

The error of the equation for different lapse rates in °C per km at the level of the maximum velocity is given by BUELL [1943] as follows: For fully developed cumulo-nimbus, the conditions usually existing during severe thunderstorms:

Lapse rate (°C) × height of level of maximum velocity (km)	30	35	40	50	60	80
Error, per cent	24	12	7	-7	-13	-23

For well developed cumulus congestus, with the anvil about to form, the errors are negative, ranging from -10 per cent for lapse rate value 30°.km to -27° for a value of λ = 80°. The errors are negative, usually -1 to -10 per cent while cumulus congestus is developing. The maximum ascent velocity by Buell's formula should be comparable with v_2, the ascent velocity at the upper end of the convection tube, as given by (25).

Ascent velocity, third method--The thunderstorm model may be considered as a hydraulic system discharging a volume V_1 of air (at initial pressure and temperature) per unit of time from one reservoir to another through a pipe of average radius r_m and length L, with a pressure difference $P_1 - P_2$ between the two reservoirs. If steady flow occurs and the resistance to flow varies as the square of the velocity, then the velocity will vary as the square root of the slope or pressure gradient and

$$v_m = k'\sqrt{P_1 - P_2/L} \quad \ldots\ldots \quad (28)$$

The factor k' depends on the radius r_m and the units used. Variation of diameter of the convection tube also affects v_m. This effect may be included in the constant k'. Also,

$$V_1 = a_m v_m = k' a_m \sqrt{P_1 - P_2/L} \quad \ldots\ldots \quad (29)$$

For pseudo-abiatic ascent the ascending air is assumed to be in equilibrium with surrounding air. The pressure difference $(P_1 - P_2)$ is the buoyancy of the moist warm surface air below the cover layer. Vapor weighs 0.623 times as much as dry air. The buoyancy of unit volume of moist air overlain by drier air is equivalent to an upward-directed force equal to the difference in weight of the two unit volumes and hence, other things equal, to the difference in vapor pressures, so that $e_1 - e_2$ may be used instead of $m_1 - m_2$. The pressure difference $P_1 - P_2$ also includes the buoyancy (if any) of the surface air resulting from surface overheating. This is proportional to the surface temperature excess above the dew-point temperature, or $P_1 - P_2 = (e_1 - e_2) + k'(\theta - \theta_d)$. The residual vapor pressure e_2 and the second term in this equation are both usually small relative to e_1 for conditions producing maximum storms except in semi-arid regions, and $k'(\theta - \theta_d) - e_2$ may be replaced by a quantity c, which can, as shown later, be determined for a given region from observed maximum rainfall data.

Making $k = k'/\sqrt{L}$ gives

$$v_m = k\sqrt{e_1 + c} \quad \ldots\ldots \quad (30)$$

There are various other effects and factors of secondary importance which are thrown into k and c and are taken into account in their determination from observed data.

For steady conditions $C_r = i_s A_s$. If i_s and A_s are known, then C_r can be determined from the rainfall data. Methods for determining i_s and A_s from rainfall are subsequently given. Then, from (14)

$$v_m = C_r/a_m m_c \quad (31)$$

The condensible moisture m_c can be determined from vapor pressure e_1. Combining (30) and (31) gives

$$k = C_r/A_m m_c \sqrt{e_1 + c} \quad (32)$$

This provides a means of determining the coefficient k in the ascent velocity equation (30).

The volumetric condensation rate is, from (14), for $\rho_e = 2.00$

$$C_r = 0.693\ m_c v_m,\ \text{in cm}^2/\text{m/sec} \quad (33)$$

This simple equation is rational in form in the sense that it is based on the equation of continuity and the law of resistance for turbulent flow. It is subject to the limitation that it applies only where the resistance to flow is proportional to the square of the velocity or where the flow is turbulent, as is unquestionably the case in a rapidly ascending air current.

Ascent velocity, fourth and fifth methods--In a storm where there is neither gain nor loss of aerial storage as rain or cloud, the precipitation rate on the rain splash area equals the condensation rate in the convection tube. If $m_c = m_1 - m_2$, is the air mass of vapor condensed out of unit volume of air entering the convection tube, and R is the radius of the rain splash over which rain is falling at a given instant, $\pi r_1^2 m_c v_1 = \pi R^2 i_a$, where i_a is the areal average rain intensity over the rain splash. This gives

$$v_1 = (R/r_1)^2 = i_a/m_c \quad (34)$$

In order to apply the last equation it is necessary to know the radius R of the rain splash. This may be determined, as shown in the subsequent sections, from data of drop sizes and their suspension velocities. Such data also provide an additional method of determining ascent velocity. This method and a comparison of the various methods of determining ascent velocity will be given later.

Terminal or suspension velocity--The terminal or constant velocity attained by a falling raindrop is the same as the ascent velocity of an air current capable of holding the drop in suspension. LAWS [1941] has reviewed various earlier determinations of the size of drops held in suspension by ascending air currents of different velocities and has also given the results of additional field and laboratory experiments. The data from Laws' experiments and outdoor observations in rainstorms are given in the form of a diagram. The results are quite accurately expressed by the following equation

$$d = 5.7 - 1.55\ (v_s)^{0.2} \sqrt{9.3 - v_s} \quad (35)$$

In this equation d is in mm and v_s in m/sec. The results are as follows:

v_s	= 2	3	4	5	6	7	8	9	9.3
Laws,	d = 0.35	0.70	0.90	1.3	1.7	2.25	3.0	4.3	5.7
Calculated,	d = 0.70	0.86	1.00	1.27	1.66	2.24	3.02	4.38	5.70

Drops larger than 5.5 mm in diameter sometimes occur but they are unstable and will break into smaller drops.

Rain spread--Moisture condensed at lower levels in the ascent of air through the convection tube is carried upward to the hail and snow stages and then, in accordance with the theory of rain formation by BERGERON [1944], the cloud particles are combined into drops as a result of the excess of vapor pressure of supercooled water over that of ice. Drops reaching diameters such that their suspension velocities exceed the ascent velocity are precipitated directly from the convection tube. Smaller drops are carried upward and outward into the outflow layer.

With notation as shown on Figure 7, a raindrop having a terminal velocity v_s in still air, if carried to the top of the outflow layer, will be transported horizontally a distance such that the time of horizontal travel equals the time Δ'_t required for the drop to fall through the outflow layer. When the drop has reached the base of the outflow layer it will be no longer supported by an ascending air current and will fall to the ground at its terminal velocity. No rain will occur from

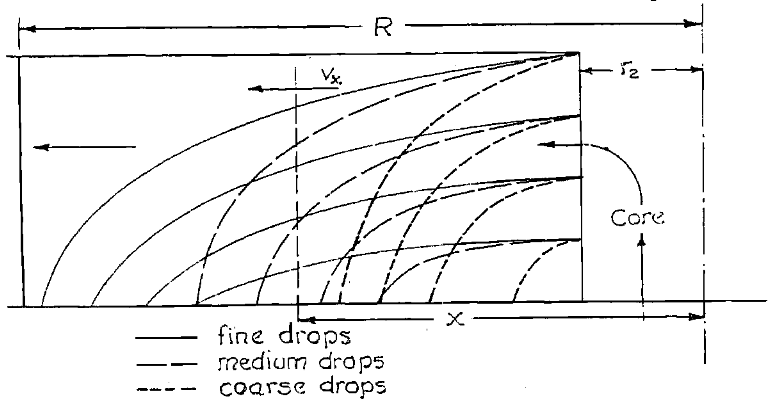

Fig. 7--Trajectories of rain drops in the outflow layer

drops of such size as to evaporate in falling from the base of the outflow layer to the ground. This minimum size of drop will be designated d_0 and may be taken as 1/2 mm diameter, since the drop-size distribution data of BENTLEY [1904], LENARD [1940], and others show that drops of less than this diameter do not usually contribute as much as one per cent of the volume of rain. For this diameter the suspension velocity is 2.3 m per sec. A drop of this size carried upward to the top of the outflow layer will be transported horizontally or radially outward in the outflow layer to a distance or radius R. The radius R fixes the boundary of the area from which rain is precipitated at a given instant. This area will be called the rain spread.

Let x represent the distance from the center of the convective core to any point in the area of the rain spread. Then, from Figure 7, for equal volumes of outflow from the core and for a cylinder of radius x and altitude Δ_2 the following relations may be obtained

$$2\pi \times \Delta_2 v_x = \pi r_2^2 v_a$$

or

$$v_x = r_2^2 v_a / 2\Delta_2 x \qquad (36)$$

Also

$$2\pi r_2 \Delta_2 v_0 = \pi r_2^2 v_a$$

or

$$v_a = 2\Delta_2 v_0 / r_2 \qquad (37)$$

and

$$v_0 = v_a r_2 / 2\Delta_2 \qquad (38)$$

Substituting v_a from (37) in (36)

$$v_x = r_2 v_0 / x$$

If the terminal velocity of drop of diameter d_0 is v_d, the time of fall through the outflow layer will be approximately $t' = \Delta/v_d$. The mean velocity v_a from r_2 to R is

$$v_a = [1/(R - r_2)] \int_{r_2}^{R} v_x dx = [v_0 r_2/(R - r_2)] \log_e (R/r_2) \qquad (39)$$

But $t' = R - r_2/v_a$ and $t' = \Delta_2/v_d$.

Equating these values of t′ gives

$$R - r_2 = (v_a/v_d)\Delta_2 = [(\Delta_2/v_d)v_0 r_2/(R - r_2)]\log_e(R/r_2)$$

and

$$(R - r_2)^2 v_d/\Delta_2 v_0 r_2 = \log_e(R/r_2)$$

Since

$$v_0 = v_a r_2/2\Delta_2$$

$$(R - r_2)^2 \, 2v_d/r_2^2 v_a = \log_e(R/r_2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (40)$$

Let $\rho = R/r_2$ and $\rho r_2 = R$. Then

$$(\rho - 1)^2/\log_e \rho = v_a/2v_d \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (41)$$

Equation (41) permits the rain splash area R to be determined from the ascent velocity v_a. This equation is not readily solved directly from ρ but values of ρ in terms of v_a are given by the line A (see Fig. 8). Line B gives values of $1/\rho^2$, which is proportional to the ratio of rain intensity in the rain spread area to condensation rate in the core. Combining these results the ratio of rain intensity to condensation rate can be expressed in terms of ascent velocity, as shown on Figure 9. The results last given relate to either a stationary storm or to instantaneous values in a moving storm.

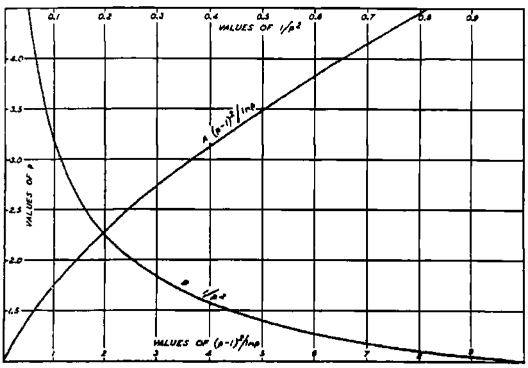

Fig. 8--Values of $(\rho - 1)^2 \log_e \rho$ and $1/\rho_2$

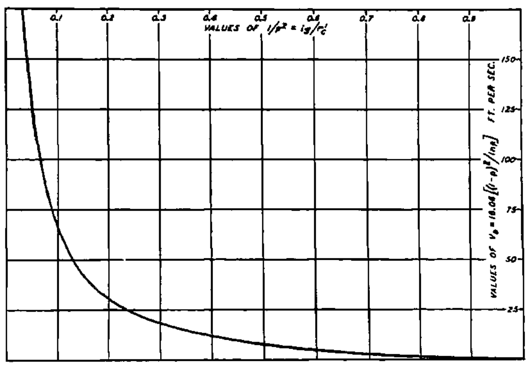

Fig. 9--Values of v_a and $1/\rho^2$

Rain intensity pattern--A raingage located in the path of the rain splash records a cross section of the storm as it passes. The ascent of air through the convection tube divides the precipitation into two components: (1) Drops with suspension velocities exceeding the ascent velocity. This component is precipitated on the projected area under the convection tube and will be called core rain. (2) Smaller drops, which are carried aloft and precipitated in the outflow layer. This is obviously from the peripheral rain belt surrounding the convection tube, and will be called peripheral rain. It is assumed that sensibly all the rain falls on a rain-splash area equal to the rain-spread area.

Segregation of rain intensity into core and peripheral components provides three main types of a rain-intensity pattern. Typical examples from heavy thunderstorms at St. Louis are shown on Figures 10 to 12. These intensity graphs show numerous rain gushes, usually of one to three minutes' duration, typical of intense thunderstorms everywhere. These rain gushes apparently result either from (1) violent wind gusts in the ascending air current or (2) swaying of the convection tube. On Figures 10 to 12 these rain surges have been eliminated by averaging the rain intensities for five-minute intervals, giving the smoothed graphs shown by dashed lines.

These smoothed graphs reveal the three main types of rain-intensity distribution in thunderstorms: (1) Graphs with a single maximum; (2) double maxima, of which the first is higher; and (3) triple maxima, of which the first is the highest. The occurrence of these maxima is related

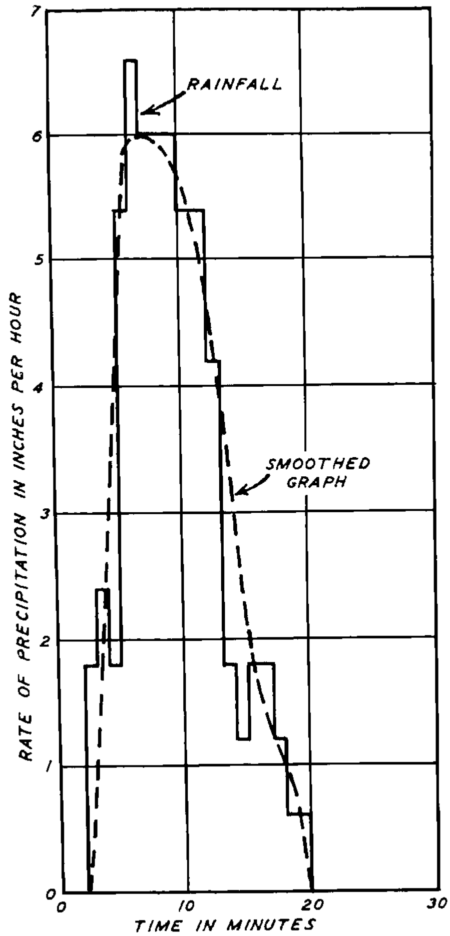

Fig. 10--Rain intensity pattern, Ewing and Washington Block, August 24, 1924

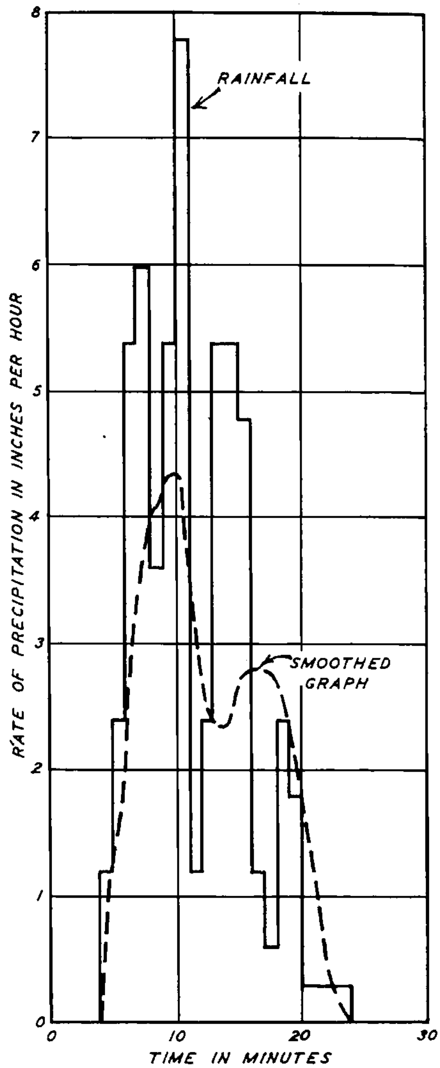

Fig. 11--Rain intensity pattern, City Block 4841, August 24, 1924

to the concentration of rain in the marginal zone between the top of the convection tube and the peripheral belt.

Two factors tend to concentrate high rain intensity in this marginal zone. The first is the concentration of larger drops in the outflow layer close to the convection tube as a result of the greater terminal velocities and consequently smaller radii or horizontal transport of these large drops. The second is the variation of velocity distribution over the cross-section area of the convection tube. For flow in a pipe the maximum velocity is at the center and is approximately 1.19 times the mean velocity, and the peripheral velocity is less than the mean velocity. More of the condensation formed in the tube near the boundary wall is precipitated than in the center of the tube, with a consequent tendency, so far as core precipitation is concerned, to concentrate higher intensities around the circumference of the tube.

Thunderstorms in which there are two rain-intensity maxima, separated by a short interval with either moderate rain or no rain, are extremely common. In the central low-intensity portion of such a rain-intensity pattern (see Fig. 13) rain falls only at such an intensity as is permitted by the ascent velocity of air in the storm core. In more intense storms this may be zero and two wholly separate rain periods result.

Rain-intensity patterns with two maxima occurred in at least 12 out of 25 heavy thunderstorms at St. Louis. Several others were on or near the border line between types 1 and 2. Only two or three were distinctly of the third type, with three maxima.

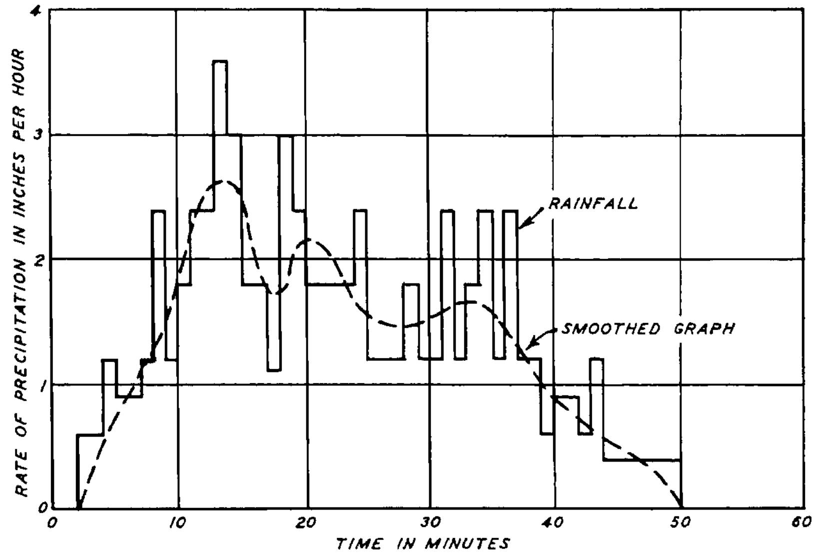

Fig. 12--Rain-intensity pattern, City Block 4841, August 12, 1916

In general the radius of the rain splash in front of the storm is less than behind the storm, the precipitation is more concentrated, rain intensity begins abruptly and reaches a maximum as the edge of the storm core reaches the station. Consequently maximum rain intensity generally occurs early in the course of the passage of a thunderstorm, commonly at about one-third the rain duration, as shown on Figure 13. For storms with two or three maxima the first is usually the highest.

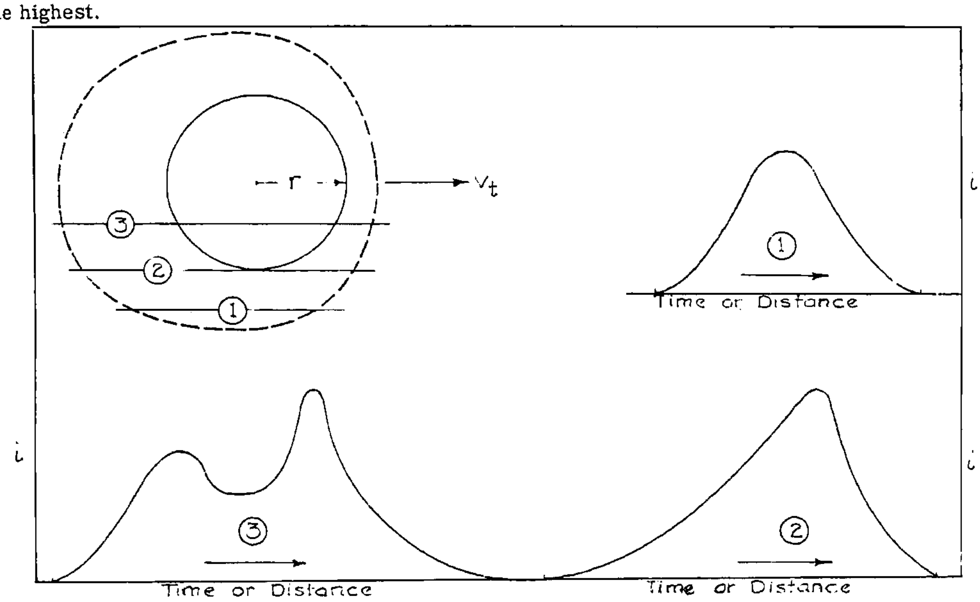

Fig. 13--Rain-intensity patterns

Figures 14 and 15 show rain-intensity patterns for the same storm for two rain gages at different locations in St. Louis. The rain gage at City Block 4841 was apparently located under the path of the core of the storm and is typical of rainfall patterns for this location. The rain gage at Ewing and Washington Block was apparently located outside but near the edge of the core of the storm and is again typical of a rain-intensity pattern for this location. The total rainfall duration was sensibly the same in both cases and both intensity patterns show the rapid variations of rain intensity resulting from gustiness.

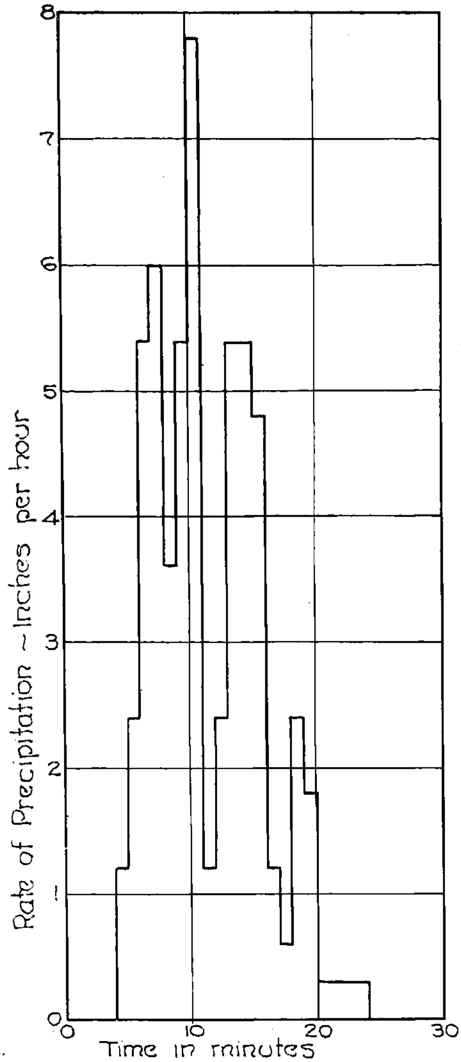

Fig. 14--Rain-intensity pattern, City Block 4841, August 24, 1924

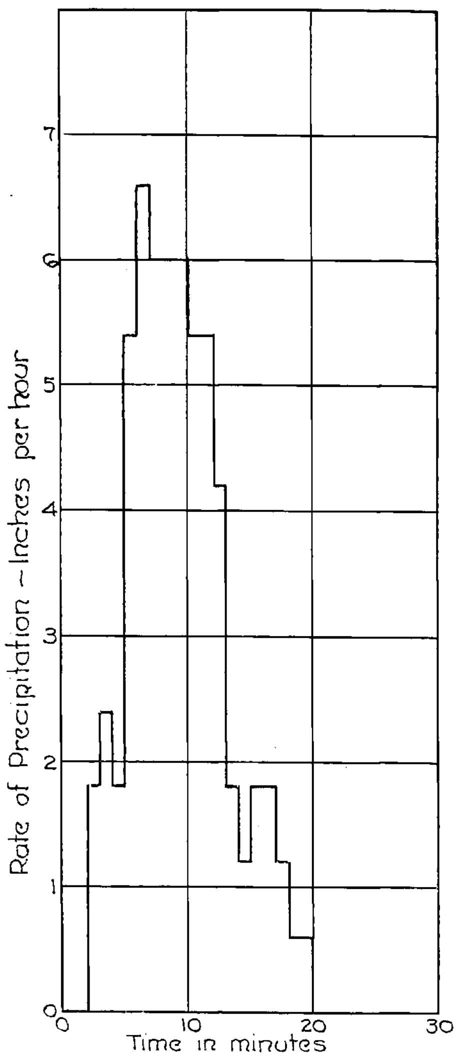

Fig. 15--Rain-intensity pattern, Ewing and Washington Block, August 24, 1924

Figure 16-A shows the average intensity pattern of 26 thunderstorms observed at the Horton Hydrologic Laboratory, near Vorheesville, New York, while Figure 16-B shows average rain intensity in 23 non-thunderstorm rains at the same station.

Intensities occurring at the same percentage of total rain duration were averaged in both cases. The graph for thunderstorms was for heavier storms nearly central over the station and this graph shows a marked central depression. The average curve closely resembles the type curves for thunderstorms with abrupt onset of rain, rapid rise to maximum rain intensity at about one-third the rain duration, and a second smaller maximum intensity following the central depression, the rain intensity then trailing off slowly to zero. The fact that the average of the many storms plotted on the percentage basis has the same characteristics as a typical individual storm shows that these different storms were much alike in rain structure, as if they were models of the same pattern but to different scales.

A rain intensity graph with a single maximum may occur in two ways: (1) With low ascent velocity. Nearly all the condensation is precipitated from the core, giving a single maximum at or near the center of the storm; (2) with a high ascent velocity. Most of the condensation within

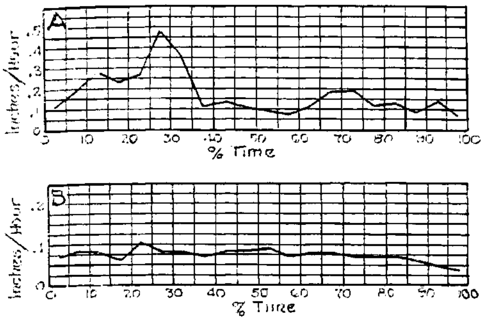

Fig. 16--Comparison of rainfall intensities; (A) thunderstorm intensity; (B) non-thunderstorm intensity

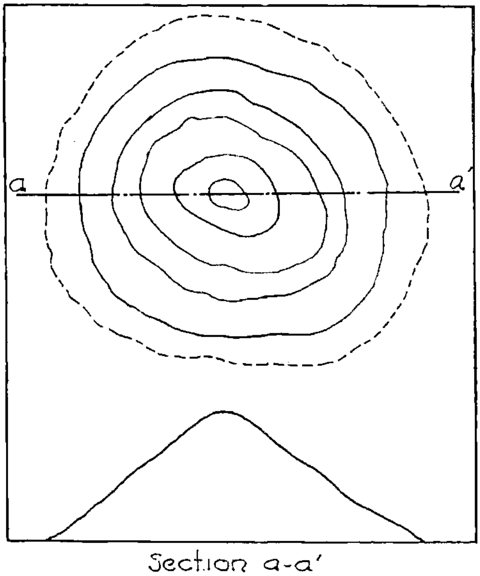

Fig. 17--Storm rain-intensity pattern

the core is carried into the outflow layer and precipitated from the peripheral belt. With respect to an observer located under the peripheral belt outside the core, as at A1, (see Fig. 13), this will produce a single-crested intensity pattern.

A double-crested intensity pattern is produced by a high ascent velocity with respect to an observer located in the path of the core as at 3 on Figure 13. These two patterns obviously merge. With respect to an observer just within the core, there would be only a slight depression of the intensity near the center.

The third, less usual, case of a rain-intensity graph with three maxima appears to be in most cases an intermediate or transitional case between type 1, single maximum, produced by low ascent velocity, and type 2, double maxima, produced by extremely high ascent velocity.

As shown by Figure 8, there is a maximum number of raindrops produced for a single drop size and ascent velocity. If the ascent velocity at the center of the core is near but less than that corresponding to this maximum drop frequency, the precipitation rate at the center of the core may greatly exceed that at the margin of the convection tube, with the result that there will be three rain-intensity maxima.

Wind gusts at the ground surface are usually of short duration, rarely lasting over one minute. Such gusts are apparently responsible, in conjunction with the sorting of raindrops by differences of suspension velocities, both in the core and outflow layer, for most short rain surges lasting from one to three minutes. There may also be longer vibrational swings of wind velocity capable of producing a maximum of rain intensity but usually much less marked than the typical maximum of rain-intensity patterns described.

Prolonged upward swing of the ascent velocity as the core of the storm reaches an observer, followed by a lull while it is passing, can produce a three-crested intensity pattern. The same result would follow if one thunderstorm overtook another, so that the two peripheral belts merge just as they passed the observer. As shown later, a three-crested intensity pattern is possible as a result of a high degree of inclination of the convection core, such that the highest core rain intensity is precipitated outside of and behind the precipitation from both the forward and back portions of the peripheral belt. The relative infrequency of well defined three-crested intensity graphs indicates that the operation of the last three causes named is infrequent.

The convection tube model,(see Fig. 4), accounts for the characteristics, relative frequency and orderliness of thunderstorm rain-intensity patterns. This orderliness can scarcely be fortuitous.

Derivation of storm characteristics from rain-intensity pattern--In addition to the methods given, much may be learned about the dimensions and velocities involved in a thunderstorm by using the rain-intensity graph as a basis. The total rainfall duration t_p at a given location within the rain spread is approximately the time of transit of the rain splash over the station. Rain duration with respect to an observer on the axial line aa' (see Fig. 17), is

$$t_p = 2R/v_t$$

while with respect to an observer at a distance z from the axial line the rain duration is

$$t_p = (2R/v_t)\sqrt{R^2 - z^2} \qquad (42)$$

For a rain gage in the path of the center of the convection tube the time t_c from the primary to the secondary rain intensity maximum is approximately the time required for the storm to travel a distance $2r_2$ equal to the top diameter of the storm core.

Using notation as shown on Figure 18, the horizontal scale represents time or distance reading to the left from a zero point at the storm front at 0. The vertical scale represents rain intensity.

From the equations

$$\left.\begin{array}{l} 2r_2 = v_t t_c \\ w_f = \text{frontal width of peripheral belt} = t_f v_t \\ w_b = \text{rear width of peripheral belt} = t_b v_t \end{array}\right\} \qquad (43)$$

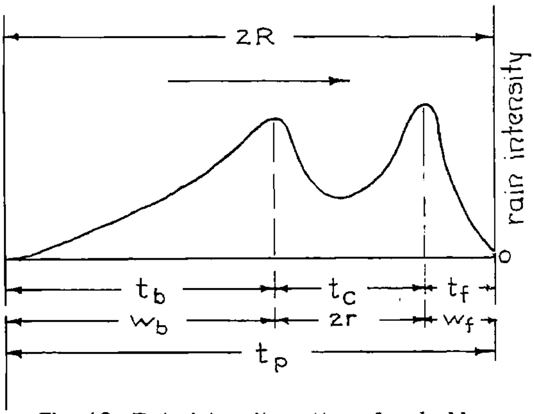

Fig. 18--Rain-intensity pattern for double-crested storm

In addition, the lateral or side width of the peripheral rain splash can be determined from the equation

$$w_\ell = (t_f + t_b)v_t/2 \qquad (44)$$

The total width of the storm path is

$$W = 2r_2 + 2w_\ell \qquad (45)$$

At a given instant rain is falling on the rain-splash area, which is taken equal to the rain-spread area, or

$$A_s = \pi R^2 \qquad (46)$$

As the storm travels, a point on the path of its center records a cross section of the areal rain-intensity pattern. The volume of rain per unit of time on the rain-splash area is C_r.

If the rain-intensity pattern is symmetrical, then this is also equal to the volume of a solid of revolution (see Fig. 17) formed by rotating one-half of the rain-intensity pattern about a vertical axis at the center of the pattern. From the theorem of Pappus or Guldinus, the equation of the volume of a solid of revolution is

$$V_R = 2\pi R_c a_p/2 \qquad (47)$$

where a_p is the area of the rain-intensity pattern. Then

$$a_p = i_a t_p \qquad (48)$$

R_c is the distance from the axis of symmetry to the center of figure of a half section. For similar figures this is a constant fraction k_R of the radius R of the rain-splash area. Combining these results gives

$$C_r = 2\pi k_R R\, i_a t_p/2 \qquad (49)$$

This equation gives the mass condensation rate C_r in terms of the quantities k_R, R, i_a, and t_p, all of which can be determined from the rain-intensity pattern.

The form of surface of the solid of revolution shown in Figure 17 corresponds to a single-crested rain-intensity pattern or to peripheral rainfall. The same procedure for determining C_r can be applied approximately for a two-crested intensity graph corresponding to the usual rainfall pattern for maximum storms, as shown on Figure 19, with unequal maxima if the vertical axis of the rainfall pattern is so located that the segments on the right- and left-hand sides of the axis are equal and the value of R_c used is the average of the values determined for the right- and left-hand segments.

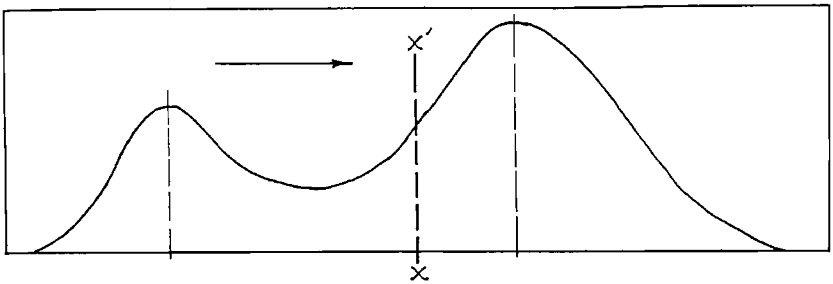

Fig. 19--Storm with two crests

Rains producing maximum splash intensity--For steady conditions in a fully developed storm, C_r is sensibly equal to the total precipitation rate. For a vertically convective storm this rain is spread over an area having a diameter

$$R = r_2 + B \quad \text{(50)}$$

where B is the average width of the peripheral rain belt surrounding the core at outflow level, or

$$B = R - r_2$$

The area of the rain splash is

$$A_s = \pi (r_2 + B)^2$$

Also, as already shown in (13)

$$C_r = \pi r_1^2 v_1 m_c \quad \text{(51)}$$

where m_c = moisture condensed out of unit volume of initial air, expressed as equivalent volume of water. If, for example, r_1 and v_1 are in m and m/sec, and m_c in cc, then C_r will be expressed in cc/sec on the area πr_1^2.

In terms of r_2 and v_2

$$C_r = \pi r_2^2 v_2 m_c/\rho_e \quad \text{(52)}$$

The areal average intensity over the rain splash area is

$$I_a = k_c C_r/A_s = r_2^2 v_2 m_c/(r_2 + B)^2 \rho_e \quad \text{(53)}$$

where k_c is a proportionality factor to convert mass condensation in the given units of volume into depth in chosen units on the cross-section area of the convection tube.

From (50)

$$B = R - r_2$$

which is a function of v_m/v_d or of v_2/v_d, which r_2 is a function of r_1 and ρ_e. For a given B there will be some value of r_2 for which I_a (see Eq. 53) will have a maximum value. If other factors are assumed constant and r_2 varies, then differentiating and equating to zero to obtain the value of r_2 for which I_g is a maximum gives

$$r_2 = B \text{ or } R = 2r_2 \quad \text{(54)}$$

or the splash intensity will be a maximum for a storm where the outflow radius of the convection tube equals the width B of the peripheral belt of outflow.

The initial radius of the convection tube, and consequently its outflow radius, is dependent chiefly on irregularities of the stable cover layer. Once formed, it may and probably does enlarge as the storm advances. If the radius r_2 is initially less than B, the value required for maximum point intensity, with the convection tube gradually enlarging, will be a certain point along the course of the storm path at which r_2 has the required value for producing maximum-splash rain intensity. There and there only will the maximum-splash intensity occur for the given conditions.

Point-rain intensity is the time intensity as recorded by a given rain gage. If this gage is on the storm path, it will record a diametral cross section of the rain-splash intensity. The point intensity for the time t_p required for the rain splash to pass over the gage will be a maximum for the same conditions for which the splash intensity is a maximum.

Limitation of core-rain intensity--Total rainfall is comprised of two components: (a) Core rain, which is immediately precipitated, and (b) peripheral rain, which is precipitated from suspension storage. The relative amounts of these components are determined for a given drop-size distribution by the average ascent velocity during the passage of the convection tube over the recording station. Conversely, the average ascent velocity can be determined from the relative proportions of the two rain components.

The condensation rate is an increasing function of the ascent velocity, whereas the fraction precipitated is a decreasing function of the ascent velocity, and their product, which represents the precipitation rate, has a maximum value for some particular ascent velocity v_m.

As a basis of calculation, air saturated at 79.7°F at entry to the convection tube has been used. This corresponds to e = 1.023 inch mercury or 25 cm^3/m^3. If 80 per cent of this is precipitable, then m_c = 20 cm^3/m^3 = 0.02 mm/m^2 depth of condensation per sec for an ascent velocity of one m/sec = 2.84 inch depth/hr on the projected convection-tube area per m per sec ascent velocity.

Using this condensation rate in conjunction with Lenard's drop-size distribution of LENARD [1940] for a severe thunderstorm, calculations were carried out as given in Table 3. Column (3) is the observed number of drops per square meter for each class size; column (4) the volume of drops of a given class size, and column (5) the percentage of total volume of rain included in drops of a given size. The drop size corresponding to the upper limit of a given class is shown in column (6) and the percentage of total volume of rain occurring in drops exceeding a given size in column (7). These results, together with corresponding results from Lenard's series of drop-distribution data are shown on Figure 20. The corresponding suspension velocities, from Laws' data, are given in column (8), and the corresponding condensation rates in column (9). The condensation

Table 3--Rain formation rates with various ascent velocities [LENARD, 1940, p. 155]

Drop diameter	Volume per drop	Number of drops per m^2/sec	Volume	Volume per cent of total	Class limit	Volume per cent larger than limit	Ascent velocity v_a	Condensation rate	Precipitation rate	
1	2	3	4	5	6	7	8	9	10	11
mm	mm^3		mm^3		mm		mps	mm/sec	mm/sec	in/hr
0.0	.....	...	....	....	0.25	100.00	1.25	0.0125	0.0125	1.77
0.5	0.066	514	33.92	0.37	0.75	99.63	3.25	0.0325	0.0323	4.57
1.0	0.523	423	221.23	2.32	1.75	97.31	4.87	0.0487	0.0473	6.68
1.5	1.77	359	635.43	6.66	1.75	90.65	6.08	0.0608	0.0552	7.80
2.0	4.19	138	578.22	6.05	2.25	84.60	7.03	0.0703	0.0594	8.40
2.5	8.19	156	1277.64	13.40	2.75	71.20	7.73	0.0773	0.0550	7.77
3.0	14.2	138	1959.60	20.60	3.25	50.60	8.27	0.0827	0.0418	5.91
3.5	22.5	0	.....	.....	3.75	0	...	.....	.....	...
4.0	33.5	0	.....	.....	4.25	.....	...	.....	.....	...
4.5	47.8	101	4827.8	50.60	4.75	.....	...	.....	.....	...
			9533.84	100.00						

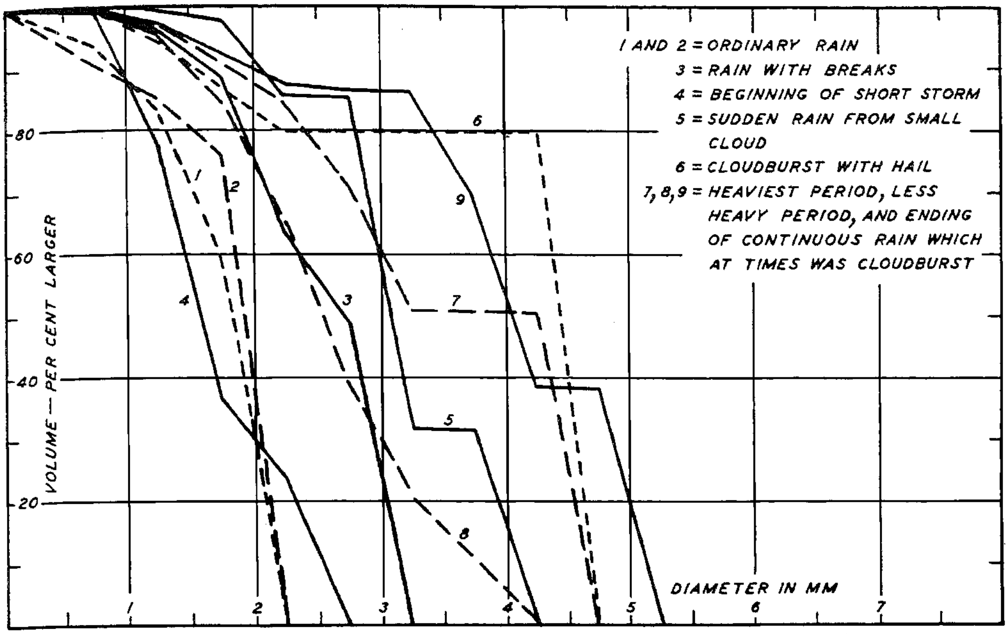

Fig. 20--Percentage of volume of raindrops exceeding a given diameter

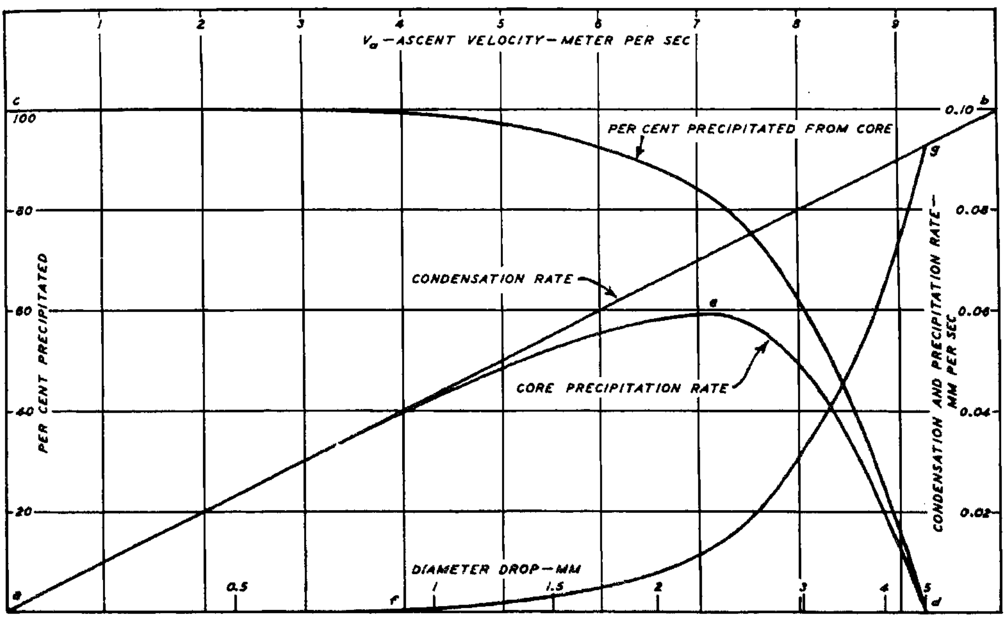

Fig. 21--Relation of condensation and precipitation rates to drop-size distribution; heavy rain period; Lenard storm

rate multiplied by the percentage of total volume having suspension velocities higher than the given ascent velocity and which are consequently able to fall through the ascending air current, represents the rate of precipitation on the projected core area. This is given in column (10). Corresponding core-rain intensities, in inches per hour, are given in column (11). These results are shown on Figure 21. The line ab is the condensation rate, cd the fraction precipitated with the given ascent velocity v_m, and the line aed the precipitation rate on the core area in mm/sec. The maximum coincident or core precipitation occurs in this case for an ascent velocity of 7.2 m/sec and the maximum core precipitation rate, as shown at the point e, is 0.06 mm depth/sec or 8.5 in/hr. This represents the maximum possible coincident or core precipitation in an ascending air current containing 20 mm of precipitable moisture per cubic m.

It will be seen from Figure 21 that for velocities of ascent v_m less than four m/sec, substantially the entire volume of rain is contained in drop sizes larger than can be held in suspension and is coincidentally precipitated, in other words, as shown by the lines ab and aed, the condensation rate and the coincident precipitation rate for v_m less than four m/sec are identical, unless larger drops are broken up and then carried aloft.

The data used for drop sizes in this study are those storms giving the largest number of large drops. Since only larger drops are precipitated from the core, with high ascent velocities, the precipitation intensity from the storm core will in general be less than that shown by this calculation if the percentage of large drops is less.

Maximum limiting peripheral-rain intensity--A similar limit exists in relation to peripheral-rain intensity. The volumetric intensity of peripheral precipitation equals the fraction of mass condensation not precipitated from the storm core. In Figure 21 the peripheral mass-precipitation intensity is the difference between the ordinate of the condensation line ab and the core precipitation line aed. This is shown by the line fg. This quantity is zero for ascent velocity less than four m/sec and increases to equality with the condensation rate at a velocity of 9.3 m/sec. As the ascent velocity increases, the fraction of peripheral precipitation increases, until, for intense storms with ascent velocities exceeding 9.3 m/sec, there is no core precipitation, a condition not infrequently observed in intense storms. The sum of the core and peripheral components of precipitation for steady conditions equals the condensation rate. As a result of gustiness the actual mass precipitation rate fluctuates above and below the coincident condensation rate.

The important results of this study are as follows:

(1) For steady conditions mass condensation and mass precipitation, expressed, for example, in cm^3/sec are equal.

(2) In case of vertical convection the precipitation intensity on the area over which condensation occurs is, except for low ascent velocities, always less than the volumetric condensation rate over the same area.

(3) Instead of occurring on the area πr_2^2, the precipitation is spread over the larger area $A_s = \pi(r_2 + B)^2$, and for maximum rain-splash intensity, $B = r_2$, the splash intensity is one-fourth the condensation rate if both are expressed in terms of depth on the area within which they occur.

Finally, as a result of a combination of (1) the separation of condensation into ascending and descending components in the storm core and (2) rain spread in the peripheral ring, vertical convection cannot produce an areal-rain intensity on the rain-splash area greater than about one-fourth the condensation rate if both are expressed in units of depth on the area on which they occur. This explains the observed fact that in convective storms, rain intensities usually fall far short of calculated condensation rates.

There are several other reasons why the rain intensity I_a is usually less than one-fourth the condensation intensity c_r: (1) R is usually greater than $2r_2$ when the maximum ascent velocity v_m occurs. (2) There may be dilution of the ascending current of moist air as the result of outside air drawn into the convection tube by turbulence, under certain conditions. (3) The convection tube may not be vertical.

Inclination of convection tube--In the analysis thus far it has been assumed that the convection tube is vertical. This will not be true if, as is frequently the case, the wind velocity w_0 in the outflow layer is different from the translational velocity v_t of the stable cover layer.

Figure 22-A and 22-B shows, respectively, elevation and plan of thunderstorm model with an inclined convection tube, it being assumed for simplicity that the convection tube has a uniform diameter $2r_m$ equal to twice the mean radius. Letters with primes will be used to designate quantities related to the inclination of the convection tube, for which the same symbols without primes apply to a vertical convection tube. For a given thickness Δ of the moist air layer and height h_t of the tropopause, the lift L will be unchanged but the length of the convection tube will be increased to L'.

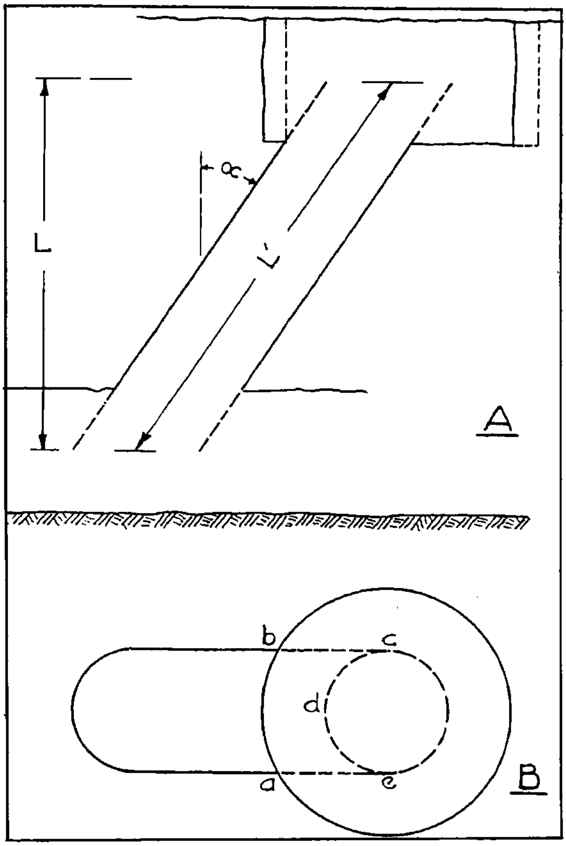

Fig. 22--Inclination of convection tube

$$L' = L \sec \alpha \qquad (55)$$

For the same pressure difference at the ends of the convection tube the hydraulic gradient is decreased by inclination of the tube from $(P_1 - P_2)/L$ to $(P_1 - P_2)/L'$, or in the ratio

$$L/L' = \cos \alpha \qquad (56)$$

(The author seems to be in error here in treating the convection tube like a pipe with a hydraulic gradient, so (56) and those that immediately follow do not apply. The driving force in a convective tube without sides is not the hydraulic gradient but the density (mostly thermal) difference between the air in the column and that outside. A leaning tube is the combined result of unchanged convective ascent and horizontal shear of the country wind. None of the force of the convection is used or lost except through such turbulence as the shear introduced in the wind. The author's computations of the reduction of areal or splash rain and the increase in core-rain intensity are accordingly too great.--C. F. Brooks)

Since the mean velocity is proportional to the square root of the hydraulic gradient, the mean velocity will be reduced from v_m to v'_m, where

$$v'_m/v_m = \sqrt{L/L'} = \sqrt{\cos \alpha} \qquad (57)$$

The volumetric condensation rate C_r is the product of $(m_c) \times$ (cross section of convection tube, measured horizontally) $\times$ (vertical component v'_v of mean velocity). The vertical component of the mean velocity is

$$\left.\begin{aligned} v'_v &= v'_m \cos \alpha \\ &= v_m \cos^{3/2} c \end{aligned}\right\} \qquad (58)$$

where v_m is the ascent velocity for a vertical tube. The horizontal cross section of the convection tube is elliptical. Its axes are $2r_m$ and $2r_m \sec \alpha$ and its area is

$$A_c = \pi r_m^2 \sec \alpha \quad \text{(59)}$$

The product of (58) and (59) is proportional to the volumetric condensation rate. This gives

$$C'_r = m_c \; r_m^2 v_m \sqrt{\cos \alpha} \quad \text{(60)}$$

The volumetric condensation rate is decreased by inclination of the convection tube in the ratio $\sqrt{\cos \alpha/1}$. For different angles the reduvtion of volumetric condensation rate is as follows:

α	= 0°	10°	20°	30°
C'_r/C_r	= 1.0	0.992	0.969	0.930

Inclination of the covection tube exceeding 30° is improbable and it will be seen that so far as volumetric condensation rate is concerned, inclination causes a small but only a small reduction. This condensation is spread over a rain-splash area which comprises the projected peripheral ring plus the part of the projected convection tube area which lies outside the ring. The projected convection tube area will lie wholly inside the peripheral ring if

$$L \tan \alpha < r_m \quad \text{(61)}$$

If $L \tan \alpha \geqq 2r_m$, an area of the projected convection tube $\geqq \pi/2 - r_m^2$ will be outside the peripheral ring. For this condition it can readily be shown from Figure 22-B that

$$A'_s/A_s = (4.5\,\pi - 4)/4\pi + L \tan \alpha/2\pi r_m \quad \text{(62)}$$

or

$$A'_s/A_s = 0.81 + L \tan \alpha/2\pi r_m = 0.81 + 0.159 \; L \tan \alpha/r_m \quad \text{(63)}$$

For example, for $L = 9.42$ km, $r_m = 1$ km, $\tan \alpha = 0.212$ and $\alpha = 12°$, (63) gives $A'_s/A_s = 1.13$ and there would be a decrease of rain-splash intensity resulting from increase of projected rain-splash area alone of 13 per cent.

Finally, for small inclinations of the convection tube the reduction in splash-rain intensity by inclination is so small as to be practically negligible, while for maximum inclinations of the convection tube ranging up to 20 to 30° the effect of increased splash area may reduce the areal or splash-rain intensity by 25 per cent or more. This result is in accordance with experience, since maximum rain intensities in the Rocky Mountain region, where there is little wind shear aloft and convection is nearly vertical, are commonly greater than for the eastern United States, with nearly the same vapor pressure but with marked inclination of the convection tube in some storms. That the inclination of the convection tube is usually small or moderate is indicated by the following considerations.

Assuming that at any given level a particle of moist air ascending with a velocity v_m takes on a horizontal component of velocity equal to that of the country air at the same level, then the convection tube will gradually increase in inclination proceeding upward, until at the outflow level, where $w_h = w_0$, the inclination will be such that

$$\tan \alpha = w_0/v_m$$

Since v_m is usually large, and in some cases extremely large, relative to w_0, the inclination angle α in most intense storms will be negligibly small.

The things compared above are the average areal intensities over the rain-splash area, while rain intensities as ordinarily compared are maximum point intensities as recorded at a given rain gage. For circular rain splash areas these two quantities, as already shown, bear nearly a constant relation to each other. If the rain-splash area is not circular but has a distorted form, as shown by Figure 22-B, as a result of inclination, the characteristics of rain distribution in the rain-splash area will be modified in the following various ways.

(1) The outflow ring may be shifted to the right, as shown by dotted lines on Figure 22-A. This will result from the increased component of horizontal outflow velocity v_0 to the right and a corresponding decrease to the left as a result of inclination of the convection tube. This will tend to

increase the ratio A'_S/A_S and to decrease the average rain-splash intensity for an inclined tube more than is indicated by (63). The increase of volume and velocity of outflow at the front and the decrease of these quantities at the back of the rain splash by inclination may be compensated by a corresponding increase of thickness of the outflow layer at the front as compared with that at the back. So far as this effect considered alone is concerned, inclination of the tube tends to concentrate the highest rain intensities at the front of the storm and to make the primary maximum rain intensity greater than the secondary maximum, as it usually is.

(2) As a result of reduction of ascent velocity by inclination, smaller drops will be precipitated from the convection core and the percentage of condensation appearing as core rainfall will be increased.

(3) Other factors are involved. Shifting of the peripheral belt in the direction of inclination tends to concentrate peripheral rain at the front. At the same time the peripheral and core rain overlap and commingle in the region abcde (see Fig. 22-B) of the rain splash. This may produce (a) a higher maximum-point intensity in the second than in the first maximum, reversing the more usual condition; (b) this produces a marked difference of drop-size distribution in the regions of the two maxima. (It is to be remembered that the rain-intensity splash as ordinarily plotted, with time reading from left to right, is a mirror image of the splash as it actually occurs with the storm traveling from left to right.)

Lightning

The preceding theoretical treatment of the operation of a model thunderstorm shown on Figure 4 provides a basis for application to the quantitative analysis of thunderstorms to the determination of their dimensions, velocities, and precipitation characteristics. Three topics closely related to the physics of thunderstorms have not been treated in detail as they are not closely related to rain production. These are: (1) Statistical distribution of drop sizes and the occurrence of dominant drop sizes; (2) rain-intensity distribution within the rain splash and over the rain smear area; (3) lightning. The first two topics are the subjects or separate papers. Both afford confirmatory evidence of the validity of the tubular thunderstorm model.

With regard to lightning, it will be assumed that the principal source of electricity of lightning is the Lenard effect or the release of electrons by the breaking up of unstable or over-size raindrops. In accordance with the author's thunderstorm model this will produce the following effects: (1) Positively charged broken drops are either slowly carried upward from the convection tube to the outflow layer or are precipitated as core rainfall, depending on the strength and succession of gusts. (2) Electrons released by the breaking of unstable drops in the convection tube will be rapidly carried upward to the outflow layer, where they will tend to remain. (3) Additional electrons will be released as the result of combination and subsequent breaking up of large drops in the peripheral belt, particularly close to the margin of the convection tube. These electrons will be carried horizontally outward in the outflow layer but will tend strongly to accumulate at the front of the outflow layer. (4) Positively charged droplets from these sources will be precipitated from the core and peripheral ring. These conditions provide (a) juxtaposition of positively charged rain descending from the peripheral ring and free electrons ascending within the convection tube; (b) negatively charged rain in the inner portion and positively charged rain or cloud in the outer portion of the peripheral ring, resulting from smaller or broken, and hence positively charged, drops traveling farthest horizontally.

A theory of the formation of lightning, presented by SIMPSON [1910] in 1906, has been subject to much criticism, chiefly on the ground that it did not provide a sufficient variety of conditions of occurrence of lightning in thunderstorms. The author's thunderstorm model provides a combination of conditions under which lightning may occur from cloud to earth, cloud to cloud horizontally, or cloud to cloud vertically, depending largely on the ascent velocity and fraction of the rain which is broken up in the convection tube and outflow layer.

Observations at Kew Observatory, by means of instruments carried on sounding balloons, have been reviewed by ROBINSON [1941]. Diagrams are given by Robinson showing a maximum frequency of positive field at height of about seven km, decreasing to zero above 12 km, also a maximum frequency of positive field close to the storm center and a still higher maximum at the extreme outer radius of the rain splash. These results are in agreement with the conditions resulting from the operation of the author's thunderstorm model.

Snow and, in particular, soft hail, are often positively electrified by friction [SIMPSON, 1942]. Also, because of its lightness, snow is carried horizontally to the outer limit of the peripheral belt.

A combination of this with positive electrification of larger raindrops in and around the storm core produces, as a statistical average, a maximum of positive electrification near the center and a stronger maximum at extreme periphery of the storm, as shown by the Kew observations, reported by ROBINSON [1941].

SIMPSON [1942] has reviewed the evidence regarding the nature of lightning which has accumulated since the original publication of Simpson's theory. He concludes, among other things, that (1) precipitation of all kinds is sometimes positively and sometimes negatively charged, (2) in all kinds of rain the amount of water which is positively charged is greater than the amount which is negatively charged, (3) positive charges on rain preponderate in thunderstorms, especially in the period of high rain intensity. Changes of electric charge are associated with (a) wind gusts, (b) abrupt changes of rain intensity, (c) changes of drop size and drop-size distribution.

The occurrence of strong and prolonged wind gusts must be taken into account in conjunction with the results of the subdivision of rain into core and peripheral components in order to arrive at a set of conditions which is comportable both with the phenomena of lightning and the observed electric fields in thunderstorms and electric charges on rain.

Lightning seems specially likely to occur when an ascending air gust in the convection tube charged with free electrons or negatively charged particles passes a descending peripheral gush of positively charged rain which has resulted from the collision and subsequent breaking up of large raindrops around the inner margin of the peripheral ring. In this way the frequent occurrence of rain gushes following violent lightning is easily understandable.

While the physics of lightning is too complex to be completely treated here, the idea that rain is subdivided into two components, (1) that precipitated from the convection core, (2) that precipitated from the peripheral belt in the outflow layer, not only leads to results consistent with observations but apparently contributes directly to a satisfactory interpretation of the observed phenomena of lightning.

Gustiness--The author's thunderstorm model assigns much importance to gustiness of thunderstorm winds both as a determinant of the type of the resulting storm and in relation to lightning phenomena. Further study of gustiness and correlation of wind gusts, rain gushes, and lightning flashes on a quantitative basis is much needed. In case of rain gushes the author and others have found that if rain intensities are plotted between each two major breaks of intensity, taking times to the nearest whole minute, a reasonably accurate detailed rain pattern is obtained. Also if the intensities are averaged for five-minute intervals the rain gushes are mostly eliminated. Using, for example, the rain pattern shown on Figure 23,

N = number of rain gushes or summits = 10
i_g = average crest intensity = 2.28 in/hr
i_0 = average trough intensity = 1.15 in/hr
i_a = average rain intensity = 1.44 in/hr
t_p = total rain duration = 48 min = 0.8 hr
Average range = $i_g - i_0 = 1.13$
Relative range = $(i_g - i_0)/i = 0.78$
Gush interval = $t_g = t_p/N = 48/10 = 4.8$ min = 0.08 hr
Gush volume = $(i_g - i_0)\, t_g/2 = 0.045$ in
Relative gush volume = $(i_g - i_0)/2\; t_g/t_p = 0.058$

If some similar method can be applied to the more complex and irregular wind gusts it would provide a basis of correlation of the two quantities.

The formation of a tubular sheet vortex in thunderstorms and hailstorms is apparently the result of air ascending through the break in the cover layer in successive gusts. (An alternative hypothesis is that these variations in precipitation rate are due to horizontal convergences and divergences in the turbulent wind.-- C. F. Brooks). The origin of these gusts presents an interesting problem.

While not qualitatively proven for air, it is well established both theoretically and experimentally that the flow of water under certain conditions cannot be steady, but initially steady flow breaks up into roll waves or into a train of roll waves [CORNISH, 1910]. JEFFREYS [1925] concluded that for the flow of water, wave trains form when the conditions are such that $v \geqq 2\sqrt{gd}$ or when the velocity for steady flow, if it existed, would be twice the velocity $\sqrt{gd}$ of a long wave in still water. This result has been roughly found experimentally by CORNISH [1925]. If a similar criterion exists in case of the formation of wave trains in air, then the conditions favorable to the occurrence of hailstorms or thunderstorms will be the combination of a relatively shallow surface layer of air and a layer of convective instability reaching exceptionally high.

The occurrence of rain in gushes in thunderstorms is a common phenomenon and the rain gushes apparently reflect the effects of individual air gusts ascending in the convection tube but are often of longer duration, commonly one to three or four minutes, whereas wind gusts accompanying thunderstorms, and neglecting minor gustiness which is superposed on the main or longer gusts, are commonly of shorter duration, usually less than one minute. Sometimes in violent

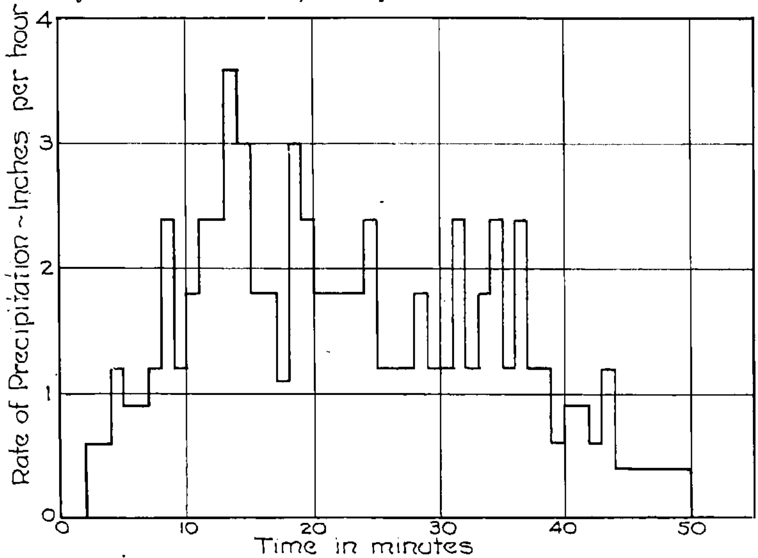

Fig. 23--Rain-intensity pattern, City Block 4841, August 12, 1916

thunderstorms strong wind gusts are repeated at irregular intervals, averaging a few seconds, although other gusts may last for several minutes. That rain gushes should be of longer duration than the wind gusts which produce them results at least in part from the fact that a wind gust entering the convection tube accelerates the upward flow of air throughout the entire length of the convection tube, although the duration of the gust may be much less than the time required for a particle of air to flow from the entrance to the outlet of the convection tube. The effect on the rain pattern is the production of a rain gush often of longer duration than the wind gush.

Thunderstorm life histories

Thunderstorms may occur singly or in groups or chains. Several simultaneous thunderstorms are attributed to widespread favorable conditions, that is, with both parents, namely, the requisite stable cover layer and the requisite underlying moist air layer, present over a wide area, with thunderstorms arising at weak spots in the cover layer, this giving rise to what may be called a "litter" of thunderstorms.

Thunderstorms may and probably do die out as a result of: (1) Exhaustion of the supply of warm moist air, particularly in case of local convective storms, or (2) the termination of suitable conditions aloft. With widespread favorable conditions the question naturally arises: Why do not the individual storms expand and merge into one enormous storm? The author has repeatedly seen two thunderstorms crossing a given line simultaneously and only a few miles apart, yet they rarely if ever merge.

It has been shown that there is a certain storm-core diameter which gives maximum rain intensity. BRANCATO [1942] reports the results of a study of over 100 non-frontal thunderstorms in the Muskingum Drainage Basin, Ohio, using the United States Soil Conservation Service network of rain gages. These storms had an average width of rain splash of about six mi, with a maximum of 12 mi. Storms originating within the Muskingum Basin usually produce appreciable rainfall for not over two hours and attain maximum rain intensity within one hour.

In case of thunderstorms, as in all cases of convection, there is a return flow of air somewhere, replacing the air flowing upward through the convection tube. As far as rain production is concerned, this return current may be considered as a gradual settling down of the air surrounding the convection tube.

The surface of discontinuity between the upflow and downflow or return current may be of at least three kinds, depending on relative velocities and density differences in and outside the core: (1) A wavy surface surrounding the core; (2) vortex rings thrown off near the boundary into the less dense fluid in the convection tube, producing ordinary turbulence; and (3) a tubular sheet vortex may separate the two surfaces. This case is considered in a separate paper. The case here considered is that of the generation of turbulence at the boundary and is analogous to turbulent flow of water in a pipe. For a given size of convection tube and a given pressure difference, $P_1 - P_2$, the ascent velocity attains such a value that Force = Resistance. The Manning formula for flow of liquids, which is dimensionally correct in form, can also be applied to air with appropriate constants to allow for difference of density. This equation may be written

$$v_m = (c/n)(r/2)^{2/3}\sqrt{(P_1 - P_2)/L} \quad \ldots \quad (64)$$

where c is a numerical constant (1.486 for water) and n is a factor proportional to the surface resistance to flow.

The convection tube has no solid boundaries and the velocity of flow relative to the walls is the sum of the ascent velocity v_m and the mean descent or return velocity v_r outside of but adjacent to the walls of the tube. The distribution of velocities in and adjacent to the convection tube is somewhat as shown by Figure 24. As the radius r increases, for the same ascent velocity v_m, the velocity v_r must increase, since the outflow velocity for a given v_m increases as the radius r increases. If the return current always extended to the same distance D from the center of the core, then the return velocity would increase in proportion to v_m. If, as is certainly the case, the distance D increases with the ascent velocity v_m, then the return velocity v_r will increase in proportion to some power of v_m less than unity but probably greater than one-half.

v_m is the ascent velocity relative to the Earth. The ascent velocity relative to the boundary of the convection tube is

$$v_b = v_m + v_r \quad \ldots \quad (65)$$

or, in terms of the Manning formula, the value of v_b is

$$v_b = v_m + v_a = (c/n)(r/2)^{2/3}\sqrt{S} = k_1 r^{2/3} \quad \ldots \quad (66)$$

where k_1 is constant for a given storm.

$$k_1 = (c/n)\sqrt{S}/2^{2/3} \quad \ldots \quad (67)$$

Substituting $v_d = k_2 r^m v$ gives

$$v_m = k_1 r^{2/3}/(1 + k_2 r^m) \quad \ldots \quad (68)$$

Differentiating and making $dv_m = 0$ to obtain the value of r for maximum ascent velocity gives

$$r_g = \sqrt[m]{1/(3k_1k_2m/2 - k_2)} \quad \ldots \quad (69)$$

In (69), r_g is the mean radius of the convection tube for which the mean velocity of ascent relative to the Earth is a maximum. Data for numerical solution of (69) are not available. The analysis, however, is strictly rational in form for the hydrodynamic model and holds true for actual thunderstorms if they reproduce the same conditions. This analysis demonstrates the existence of a convection-tube radius for which the ascent velocity is a maximum. This may or may not be precisely the radius for which the rain intensity is a maximum but will not differ greatly from it.

Without going into detailed analysis it can readily be shown that there is also a maximum rain-splash radius R. The four maxima involved, namely, maximum ascent velocity, maximum rain intensity, maximum mass-condensation rate C_r, and maximum storm radius R, are closely related but not identical although they apparently occur at about the same stage of development of the thunderstorm.

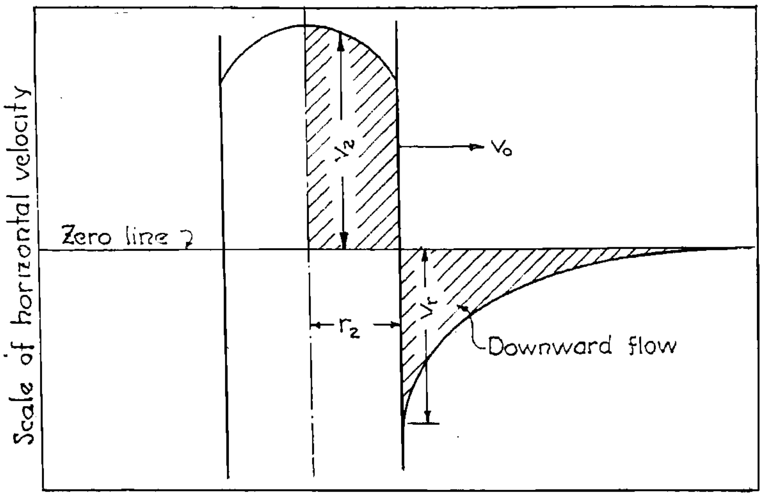

Fig. 24--Velocity distribution adjacent to outflow from convection tube (schematic)

The resistance resulting from falling rain is also a factor in limiting growth and diameter of thunderstorms. For large ascent velocities the rain is mostly peripheral and the most intense rain is concentrated close around the walls of the convection tube as a result of the sorting of raindrops by horizontal flow in the outflow layer. This occurs to an increasing extent as the diameter of the storm core increases, since the rain production for a given ascent velocity increases as the square of diameter of the convection tube, while the circumference of the convection tube varies directly as the radius. Other things equal, the intensity of peripheral rain close around the convection tube increases as the radius r. Descending rain produces a descending air current and operates in the same way as the convective return current in limiting the size of the storm. Since each of the quantities v_m, i_g, C_r, and R has a maximum value, a thunderstorm starting with a small core radius will increase in diameter and intensity until these various maxima are attained. Thereafter the ascent velocity and energy of the storm decrease and with it the outer radius R, until the storm becomes too weak to be longer sustained.

This in part accounts for the fact that storms in a different group or chain do not merge, at least they do not merge if the distance between their centers is greater than the sum of their maximum radii. However, there is another factor involved, which will next be considered.

The entrance to and outflow from the convection tube constitutes a sink and source. If two storms are near each other, then there will be two adjacent sinks at the surface and two adjacent sources at the outflow level. Two sources or two sinks tend to repel each other if their fields overlap. Two equal storms traveling in adjacent parallel paths will tend to remain at a distance from each other at least equal to the sum of their radii of influence. This radius may be determined as follows. The radial lines of flow around the outflow or source will be essentially the same as those of an electric or magnetic field where the source moves relative to the field. A more obvious analogue is afforded by the lines of flow around a well in a moving ground-water field. The lines of flow around a source are similar to those around a sink but with their directions reversed. Figure 25 shows the lines of flow around a source, moving forward with respect to its field, corresponding to outflow in a case where the motion of the cover layer in a storm is greater than that of the sub-stratosphere layer. The heavy dashed line shows the boundary of the outflow stream from the source. At an infinite distance behind the source the two branches of this boundary line become asymptotic to two parallel lines separated by a distance W. The limiting width W of the outflow stream is such that the volume of flow in the outflow layer in the width W equals the volume of outflow from the source. Within the dashed boundary line and in front of the source the flow lines curve backward, as shown. (A method of plotting these lines for a ground-water field with laminar flow has been described by SLICHTER [1905].)

It can readily be shown that the equation of the line forming the boundary of the outflow is, in radian measure

$$x = y \tan \pi (1/2 - 2y/W) \quad \ldots\ldots (70)$$

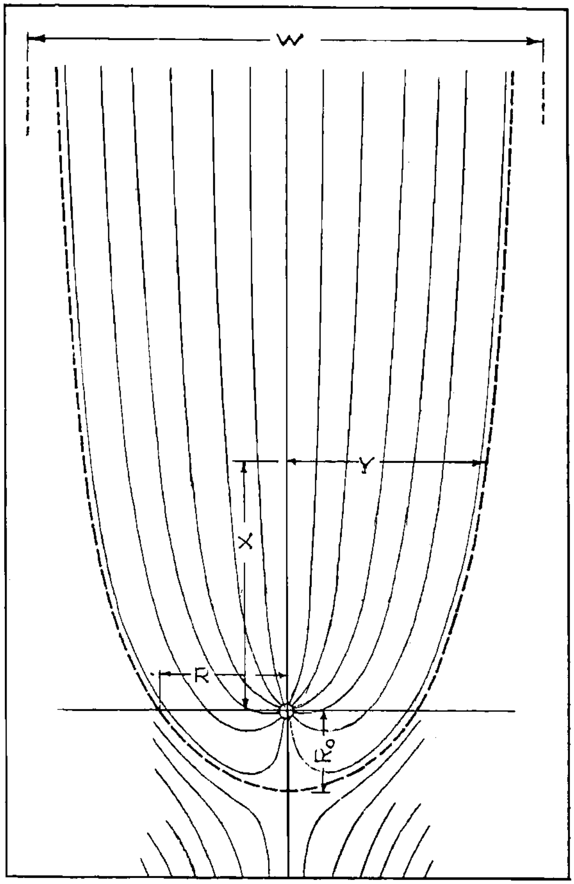

Fig. 25--Lines of outflow from a source in motion relative to its field

The width of the outflow at x = 0 or at the source is

$$2R_1 = W/2 \quad \ldots \quad (71)$$

and directly in front of the source

$$R_0 = W/2\pi = 0.159\ W \quad \ldots \quad (72)$$

From the preceding considerations it will be seen that the boundary line of the splash will be either the boundary of the outflow layer or the boundary defined by the precipitation radius R, whichever is smaller. If the boundary of the outflow layer lies inside of R, as is apparently often the case at the front of the storm, precipitation may begin abruptly. At the sides precipitation may end abruptly at a certain distance W/2, so that an observer nearly underneath this boundary may observe rain from suspension storage persisting steadily for some little time on one side of a pond, with none on the other side, or even rain on one side of a highway, with none on the other side.

The hydrodynamic model of a thunderstorm fully accounts for the observed facts regarding the life histories of thunderstorms and the fact that they follow parallel courses without merging, without assuming that the elimination of one of the two controlling conditions, that is, cover conditions and heated moisture layer, although disappearance of either one or both of these conditions affords additional reasons for the dying out of thunderstorms.

Thunderstorms, like most natural phenomean, are definitely ordered, in accordance with physical laws, yet their phenomena and life histories are almost infinitely varied.

APPENDIX I

Notation

Unless otherwise specified, foot-second units are used for flow conditions, and inches per hour for condensation and rainfall rates. Special symbols are defined where used in the text.

Subscripts 1 and 2 refer to conditions at inlet and outlet ends of convection tube.

Subscripts c, f, and b refer to conditions at center, front, and back of storm core or outflow area, as specified.

A	Area of rain smear or total area on which rain falls in a given storm
a	Cross-section area of convection tube
a'	Acceleration of the air caused by the falling rain
α	Angle of inclination of axis of convection tube to the vertical
A_p	Projected area of convection tube
A_s	Area of rain spread
B	Width of peripheral rain-belt in outflow layer
C_a	Condensation rate on projected area of convection tube
C_r	Total condensation rate in convection tube, volume per unit of time
c_m	Condensation rate on mean section of convection tube, in/hr
c_r	Condensation rate on inlet section of convection tube, in/hr
c'_r	Condensation rate in convection tube, expressed as depth on cross-section area at outflow level
Δ_1, Δ_2	Thickness of inflow and outflow layers, respectively
d	Diameter of raindrop, in millimeters or as specified
d_0	Minimum diameter or precipitated droplets
e	Vapor pressure, mb or inches of mercury, as specified
h_c	Height of condensation level
h_t	Height of base of stratosphere
θ_d	Dew-point temperature, °F
θ_K	Absolute temperature, Kelvin
i	Rain intensity for time unit, specified
i_{av}	Average station rainfall for a time interval
i_g	Maximum station intensity for time unit, specified
i_s	Average rain intensity at a given moment over the rain-splash area
J	Barometric jump, mb
L	Lift = vertical length of convection tube, center of inflow layer to center of outflow layer
L'	Actual length of inclined convection tube
m	Moisture content at saturation, gm/m^3
m_c	Condensible moisture, gm/m^3
P	Barometric pressure, mb
p	Total rainfall at a given station
p_0	Total rainfall at the eye or point of maximum rainfall in the rain smear
R	Radius of a circle having a cross-sectional area equal to that of the rain splash
R_g	Gas constant
r_1, r_2	Radius of a cricle having a crosss-sectional area equal to that of base and top of convection tube, respectively
r_m	Mean radius of convection tube
r_0	Radius of a circle having an area equal to that of the rain smear
ρ_1	$= R/r_1$; $\rho_m = R/r_m$
ρ_e	Expansion ratio
T	Time of transit of rain splash over station
T_K	Absolute temperature, Kelvin
T_s	Surface air temperature, Kelvin
t_1, t_2	Times of occurrence of corresponding phenomena at front and back of convection tube as it passes an observer on the axial line, that is, maximum wind velocitiies, or maximum rain intensities
t_p	Rainfall duration
V	Volume
v_1, v_2	Ascent velocities at entrance to and exit from convection tube, taken as center of inflow and outflow layers
v'_d	Terminal velocity of drops of minimum size

v_f, v_b	Horizontal velocity of flow across wall of convection cylinder at inlet level, front and back
v_g	Maximum ascent velocity in core
v_h	Horizontal velocity across wall of convection tube at height $\Delta/2$
v_m	Mean velocity of upward flow in convection tube
v_0	Average velocity of outflow through the walls of the convection tube in the outflow layer
v_s	Suspension velocity
v_t	Translational velocity of core of storm
v_x	Velocity at radius x in core or outflow
w	Wind velocity, relative to Earth, mi/hr
w_b	Ground wind velocity, back of convection tube
w_c	Horizontal wind velocity at center of core
w_f	Ground wind velocity, front of convection tube
w_0	Wind velocity at outflow level
x	Mixing ratio, also for radius as indicated by context

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