percentage-factor cannot be applied with a high degree of accuracy for the determination of the amount of interception. Each type of storm should be treated individually and should not be grouped.

The author's figures of 0.03- to 0.05-inch interception per storm are believed to be too conservative for snow in the Upper Connecticut Valley. The interception--measured after the snow had blown from the crowns--during a storm which occurred on April 11, 1942, in Central Vermont was 0.08 and 0.36 inch of water, respectively, in fully stocked 60-year northern hardwood and 30-year red-spruce. The amount of water which fell in the open was 0.82 inch. The interception for fully stocked pine of similar size would have been somewhere between that of the spruce and hardwood.

SIMPLIFIED METHOD OF DETERMINING AN INFILTRATION-CAPACITY CURVE FROM AN INFILTROMETER-EXPERIMENT

Robert E. Horton

Notation--

δ_a = average depth of surface-detention, including depression-storage, inches on the plat-area
δ_n = average net depth of surface-detention, excluding depression-storage, inches on area
f = infiltration-capacity, inches per hour
i = rain-intensity, inches per hour
q_s = runoff-intensity, inches per hour
t = duration of rainfall, hours
t_r = duration of residual runoff, hours
V_d = depression-storage, inches depth

The $(i - q)$-curve--The mass-line method of determining the points on an f-curve--for the sake of simplicity an infiltration-capacity curve is here described as an "f-curve"--developed by the author [see 1, 2 of "References" at end of paper] and heretofore used in the analysis of several series of infiltrometer-experiments, is simple in principle and gives accurate results. Its application is, however, laborious and somewhat involved and there seems to be a reluctance toward using it on the part of some soil-technologists and others. This reluctance may not arise from a desire to avoid labor but from the fact that calculations of f by this method are of the nature of hydraulic or hydrologic calculations, and consequently unfamiliar. This unfamiliarity with fundamental physical and hydrologic laws and processes is revealed by the substitution in some cases of what is called an "(i - q)-curve," which represents the result of subtracting surface-runoff intensities q_s, as determined by infiltrometer-tests, from the corresponding rain-intensity i, and platting the results in terms of time. The resulting curve is sometimes labeled "infiltration-capacity", which it is not. Whether so labeled or not, it is often used as if it was identical or sensibly identical with a true f-curve.

From the fundamental law of continuity of physics, commonly called in hydrology the storage-equation, $(i - q_s) = (f + d\delta_a)$.

The storage-equation gives

$$(dP/dt) - (dQ_s/dt) = (dF/dt) + (d\delta_a/dt) \qquad (1)$$

where P, Q_s, and F are, respectively, the total rainfall, surface-runoff, and infiltration, for a time-interval t. The four items of this equation represent, respectively, rates of rainfall, surface-runoff, infiltration, and change of storage or surface-detention. Letting i, q_s, f, and $d\delta_a$ represent these rates,

$$(i - q_s) = (f + d\delta_a) \qquad (2)$$

It will be seen at once that what the (i - q)-curve really represents is the infiltration-capacity f plus the rate of increase of surface-detention. The assumption that (i - q) is sensibly the same as f is, therefore, equivalent to assuming the miracle of surface-runoff occurring at a rate q_s with zero-depth of surface-detention to produce it. Actually while the net depth δ_n of surface-detention in cases of small depression-storage V_d may be comparatively small--commonly of the order of 0.1 inch or less on an infiltrometer-plat--the rate of accumulation of surface-detention is not necessarily small--in fact, at the very beginning of runoff it is equal to (i - f). For example, if i = 1.5 inches per hour (iph), which is about as small a value of i as is commonly used

in infiltrometer-experiments, and if at the beginning of runoff, $f = 1$ iph, then $(d\,\delta_a/dt) = 0.50$ iph. The value of $(i - q)$ is in this case 1.00 iph, and if this is taken as f, an error of 0.5 iph or 33-1/3 per cent results. It is only when runoff and infiltration-capacity have become constant that $(i - q)$ becomes equal to f.
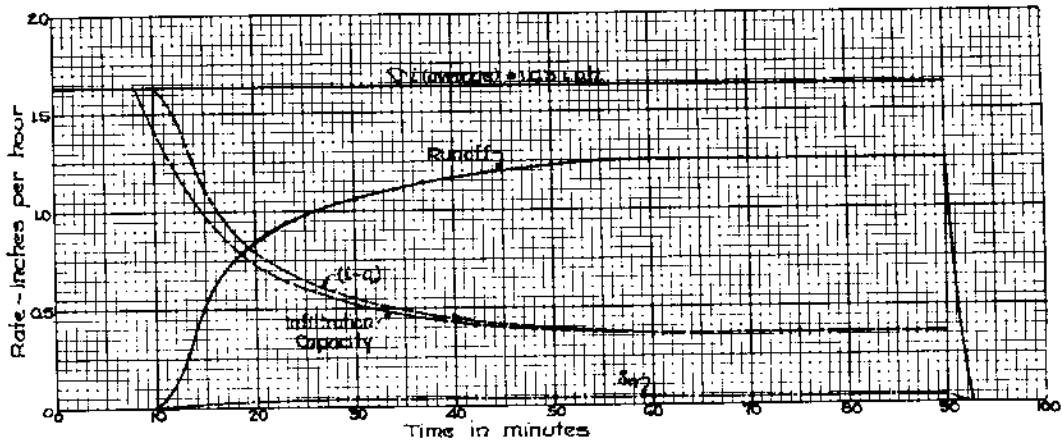

Fig. 1--Relation of f-curve and (i-q) curve from infiltrometer experiment on Marshall silt loam, bare cultivated (data from Duley and Kelly, Circular 608, U.S.D.A., Fig. 3, p. 4)

Two examples of the difference between $(i - q)$-curves and true f-curves are shown by Figures 1 and 2. In these two cases the maximum differences between $(i - q)$ and the true f-curve at the beginning of runoff are, respectively, 0.28 iph or 28 per cent on Figure 1, and 1.10 iph or 55 per cent on Figure 2. The differences decrease to zero at the time t_c at which the infiltration-capacity becomes constant. These and numerous other examples show that the use of $(i - q)$ as if it represented infiltration-capacity leads to values of f in excess of the true values in the early stages of runoff commonly ranging from 25 to 50 per cent.

Simplified method of deriving an f-curve--The $(i - q)$-curve cannot be substituted for the f-curve if anything like reasonable accuracy is to be attained. The $(i - q)$-curve may, however, serve a useful purpose in conjunction with the residual-runoff graph of an infiltrometer-experiment in deriving the f-curve directly in a simple manner.

As elsewhere shown [3], during residual runoff, infiltration takes place at capacity-rate f from a part of the area, decreasing from the whole area at the start, to zero at the end of residual runoff, in such a manner that the average area from which infiltration occurs during the residual-runoff interval t_r is ordinarily about one-third the total area, or the total infiltration during residual runoff is

$$F_r = ft_r/3 \tag{3}$$

In this equation f is the infiltration-capacity at the beginning of residual runoff, and in case the experiment lasts long enough so that f becomes equal to f_c, the latter is used.

Referring to Figure 3, the residual runoff Q_r is the area oab. If the rainfall had ended at the time t, then the residual-runoff graph would be the arc p'b of the actual residual-runoff graph transferred to cd. Both the residual runoff and residual infiltration are derived from the net surface-detention remaining on the ground at the end of rainfall. The water in depressions enters the soil as infiltration after runoff ends and need not be considered here.

The net surface-detention at the end of rainfall is
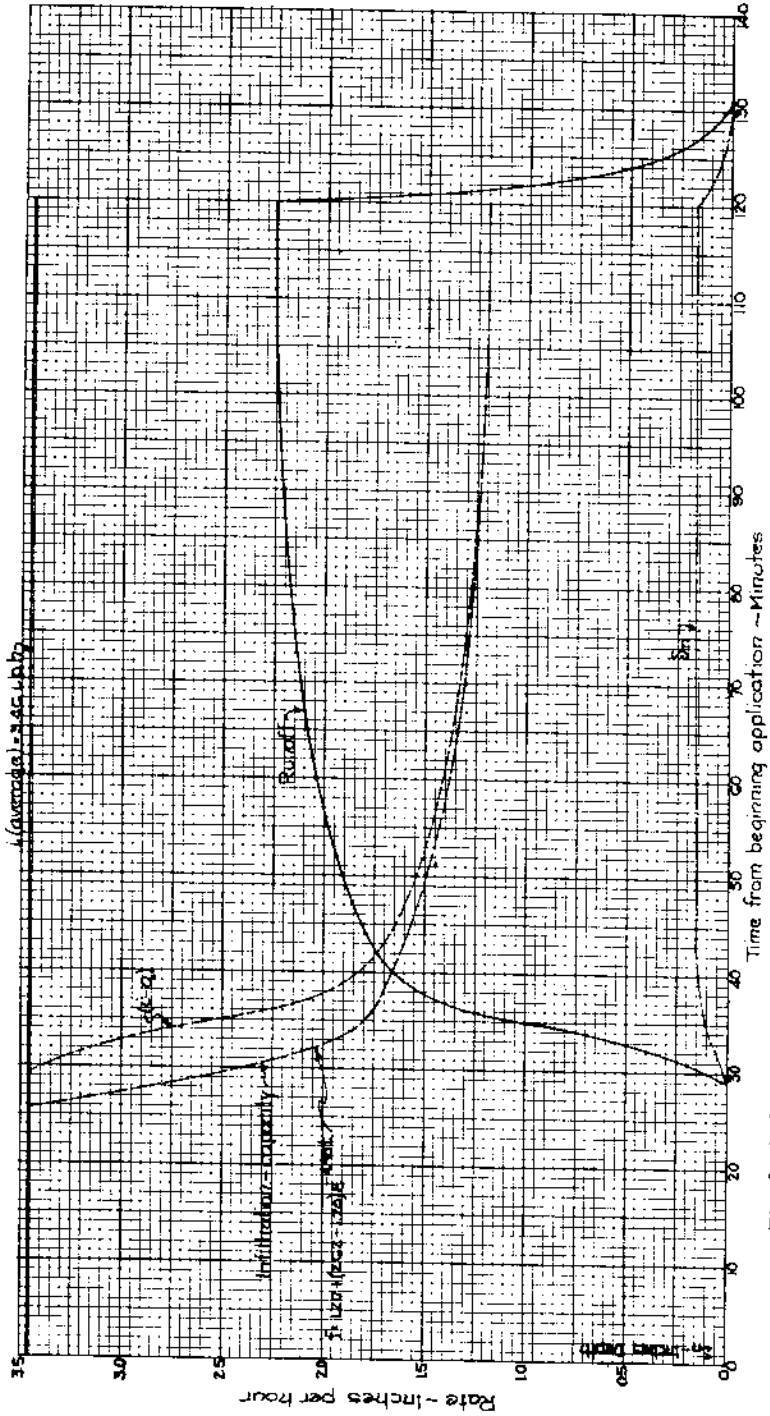
$$\delta_n = Q_r + F_r \tag{4}$$

Fig. 2--Relation of f-curve and (i-q) curve from infiltrometer experiment on Marshall silt loam with straw cover (data from Duley and Kelly, Circular 608, U.S.D.A., Fig. 5, p. 6)
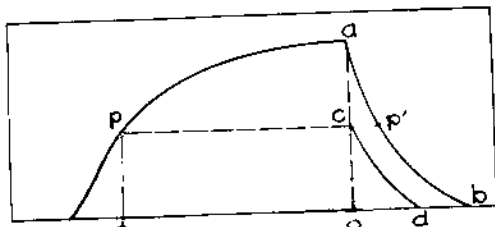
Fig. 3--Residual runoff-graph for rain ending at different times

Since the $(i - q)$-curve represents values of $(f + d\delta_a)$, values of f can be obtained by computing a series of values of δ_n by means of equation (4), obtaining graphically the corresponding values of $d\delta_n$, and subtracting these quantities from the ordinates of the $(i - q)$-curve. The best procedure is to first determine a relation-line between δ_n and q_s in the following manner.

Starting at any time t after runoff begins, project the point p on the runoff-curve horizontally to p' on the residual-runoff curve, Fi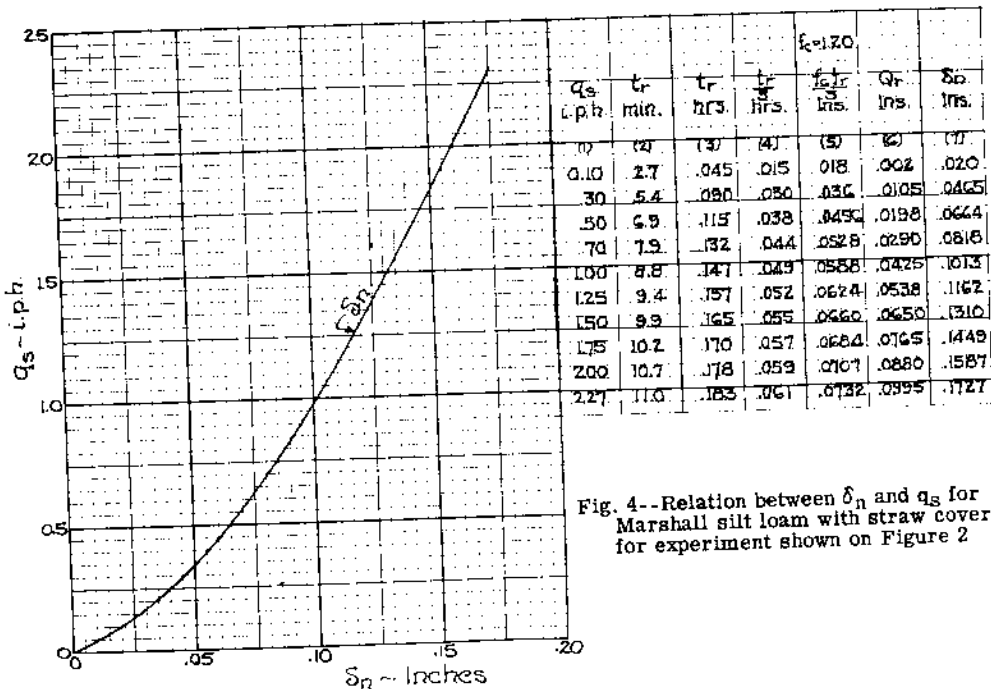gure 3. Determine the area ocd under the residual-runoff graph (arc p'b = arc cd) This expressed in inches depth, is Q_r as it would have been if rain had ended at t. Also compute the value of F_r for the time-interval $t_r = od$, t_r corresponding to time t, by means of equation (3) and find the value of δ_n by means of equation (4). A series of values of δ_n so derived can be platted in terms of the corresponding values of q_s, thus deriving a relation-curve between δ_n and q_s.

Then taking off from the hydrograph the values of q_s at the ends of successive time-intervals $(t - t')$, $(t' - t'')$, and so on, and tabulating opposite these the corresponding values of δ_n derived from the relation-curve described, a basis is obtained for evaluating $d\delta_n$. Values of $d\delta_n$, in inches per hour, are obtained by subtracting the value of δ_n at the beginning of any time-interval from its value at the end of the same time-interval and multiplying this by the factor $1/\Delta t$; the products represent approximately the values of $d\delta_n$ at the mid-points of successive time-intervals. These values subtracted from the values of $(i - q)$ at the corresponding times give a series of ordinates of the true f-curve. The computations and graphs used in deriving the f-curve on Figure 2 in this manner are shown on Figure 4 and Table 1. The inserted table on Figure 4 gives details of the

q_s c.p.h.	t_r min.	t_r hrs.	$\frac{t_r}{3}$ hrs.	$\frac{f_c t_r}{3}$ ins.	Q_r ins.	δ_n ins.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.10	2.7	.045	.015	.018	.002	.020
30	5.4	.090	.030	.036	.0105	.0465
.50	6.9	.115	.038	.045?	.0198	.0664
.70	7.9	.132	.044	.0528	.0290	.0818
1.00	8.8	.147	.049	.0588	.0425	.1013
1.25	9.4	.157	.052	.0624	.0538	.1162
1.50	9.9	.165	.055	.0660	.0650	.1310
1.75	10.2	.170	.057	.0684	.0765	.1449
2.00	10.7	.178	.059	.0107	.0880	.1587
2.27	11.0	.183	.061	.0732	.0995	.1727

$f_c = 1.20$

Fig. 4--Relation between δ_n and q_s for Marshall silt loam with straw cover for experiment shown on Figure 2

computation of the relation-curve of q_s to δ_n. Columns (2), (3), (4), (5), and (6) contain details of computation of F_r and Q_r for different values of q_s taken from the hydrograph. The corresponding values of $\delta_n = (F_r + Q_r)$ are given in Column (7) and these values platted in terms of q_s are shown by the δ_n-line on the diagram.

Table 1--Computation of f-curve

[Data from Duley and Kelly (Circ. 608,
U. S Dept. Agric., Fig. 5, p. 6)]

Time, t	q_s	δ_n	$\Delta S_n \times$ (60/t)	Mean (i - q)	f	Mid-point, t
(1)	(2)	(3)	(4)	(5)	(6)	(7)
min	iph	in	iph	iph	(5) - (4)	min
0	0	0	0	.....	.....	....
29[a]	0	0	0	.....	.....	....
32.5	0.44	.060	1.039	3.30	2.27	30.75
35	0.98	.100	.960	2.77	1.81	33.75
37.5	1.47	.129	.696	2.20	1.50	36.25
40	1.65	.140	.264	1.90	1.64	38.75
42.5	1.75	.145	.120	1.77	1.65	41.25
45	1.81	.148	.072	1.68	1.61	43.75
47.5	1.86	.151	.072	1.63	1.56	46.25
50	1.90	.154	.072	1.57	1.50	48.75
55	1.98	.158	.048	1.51	1.46	52.50
60	2.04	.161	.036	1.44	1.40	57.50
65	2.08	.163	.024	1.39	1.37	62.50
70	2.12	.166	.036	1.35	1.31	67.50
80	2.18	.168	.012	1.30	1.29	75.
90	2.22	.170	.012	1.26	1.25	85.
100	2.24	.171	.006	1.23	1.22	95.
110	2.27	.173	.012	1.21	1.20	105.
120[b]	2.27	.173	0	1.20	1.20	115.
131[c]	0					

[a]Runoff began.   [b]Application ended.   [c]Runoff ended.

Details of computation of the f-curve are shown on Table 1. The value of q_s from the hydrograph at the end of each time-interval shown in Column (1) is given in Column (2). The corresponding δ_n read from Figure 4 is given in Column (3). Then δ_n, which is the increment of δ_n for a unit time-interval, is obtained by subtracting δ_n on the preceding line from that on a given line and multiplying by (60/t), where t is the time-interval in minutes. This gives δ_n in inches per hour at a time corresponding to the mid-point of the given time-increment. In this case i = 3.44 iph and the mean (i - q) for the time-increment, Column (5), is obtained either from the (i - q)-curve or computed from i and from the values of q_s in Column (2), Table 1. The values of d_n, Column (4), subtracted from the corresponding values of (i - q), Column (5), give values of f at the mid-points of the respective time-increments or at the time shown in Column (7). The f-curve platted from these points is shown on Figure 2 and its equation given.

References

[1] ROBERT E. HORTON, Surface runoff phenomena, Publ. 101, Horton Hydro. Lab., Voorheesville, New York, February 1, 1935.
[2] ROBERT E. HORTON, Derivation of infiltration-capacity curve from infiltrometer experiments; Off. Land Use Coord., Flood Control, U. S. Dept. Agric., Washington, January, 1942.
[3] ROBERT E. HORTON, Analysis of runoff-plat experiments with varying infiltration-capacity, Trans. Amer. Geophys. Union, 1939, pp. 693-711.

Voorheesville, New York

DISCUSSION

R. A. HERTZLER (U. S. Forest Service, Washington, D. C.)--Dr. HORTON has presented a simplified useful method of deriving infiltration-capacity curves from infiltrometer-data. It should be emphasized that (i - q)-curves are different from f-curves and that the use of (i - q)-curves to calculate surface-runoff is basically incorrect. The errors in surface-runoff due to the

misuse of (i - q)-curves will be relatively large or insignificant depending upon the shape of the curve and the rainfall-pattern to which it is applied. In calculating runoff from either (i - q)- or f-curves the placement of the curve upon the rainfall-rate diagram may result in larger errors than those inherent in the approximate curves.

To calculate runoff accurately by any method is a complex, time-consuming job and the utilization of infiltrometer-data as an independent measurement of surface-runoff adds to the complexity of the calculations, but pays dividends in increased accuracy. Some time ago we analyzed a series of infiltrometer-experiments and our reluctance to using Dr. HORTON'S mass-line method as described in another publication was due to the amount of work required. To reduce the work, L. K. REINECKE modified this method and developed a procedure similar to that presented this morning. This experience with similar computations shows that Dr. HORTON'S simplified method is adaptable to "mass-production" determinations by relatively unskilled workers, and that it gives good results.

Dr. HORTON'S analysis of the phenomenon of infiltration is logical and is in accordance with field-observations obtained during a large number of actual tests. His method for determining infiltration-capacity curves involves two approximations. The first, substituting the f_c-rate in the equation $F_r = (f\ t_r/3)$, introduces an error in the subsequent calculation of δ_n, but as the differential, $d\delta_n$, is used to determine f this error nearly cancels and the resulting f-rates are substantially the same as those derived by using the calculated f, in place of f_c. This can be verified by treating the f-curve as a first approximation, reading from it the f for the given time, and inserting this rate in the member $(f\ t_r/3)$.

The second approximation, using δ_a and δ_n more or less interchangeably, ordinarily introduces no error because the depression-storage usually is about the same at the beginning of runoff as it is at the end of rainfall. This being true, depression-storage is the same on the rising and falling side of the hydrograph. There are frequent infiltrometer-tests, however, for which these depths are different. The large volume of depression-storage on irregular ground-surfaces such as exist under forest-cover may be so distributed that most of it occurs on the upper end of the plat. Under these conditions the storage-values calculated for the rising side of the hydrograph are inaccurate.

These two approximations, namely, the use of f_c and the substitution of δ_n for δ_a, cause a scatter of the derived points for the early part of the infiltration-capacity curve. The scatter reduces with increasing time so that a smooth curve, drawn through the points that do fall in line, can be extended backward through the scattered points to define an accurate infiltration-capacity curve.

We must realize that the storage-volumes on the plats have been derived in the past from measurements of rainfall and runoff only, and that an independent measurement of these volumes would extend our knowledge of infiltration-capacity. Research along this line is needed for a more thorough understanding of infiltrometer-data and data from small runoff-plats so that these may be applied more frequently in independent calculations of surface-runoff as a part of total runoff.

A SIMPLIFIED METHOD OF DETERMINING THE CONSTANTS IN THE INFILTRATION-CAPACITY EQUATION

Robert E. Horton

Elsewhere [see 1 of "References" at end of paper] the author has given a method for determining the infiltration-capacity curve or f-curve from an infiltrometer-experiment with constant rain-intensity, and has also shown [2] that such infiltration-capacity curves can be represented accurately by the equation

$$f = f_c + (f_0 - f_c)\ e^{-K_f t} \tag{1}$$

where f_0 = initial infiltration-capacity in inches per hour, f_c = final, constant infiltration-capacity in inches per hour, f = infiltration-capacity in inches per hour at time t in hours, and K_f is a factor which determines the rate of change of infiltration-capacity during rain. This equation expresses the march of infiltration-capacity during rain, for a given rain-intensity and drop-size, in terms of the three constants f_0, f_c, and K_f.

There is another important constant, t_c, which expresses the time in hours required for

infiltration-capacity to drop from its initial value, f_0, to within one per cent of its constant or asymptotic value, f_c. The value of t_c is given by the equation

$$t_c = (1/K_f) \log_e [100 (f_0 - f_c)/f_c]$$ (2)

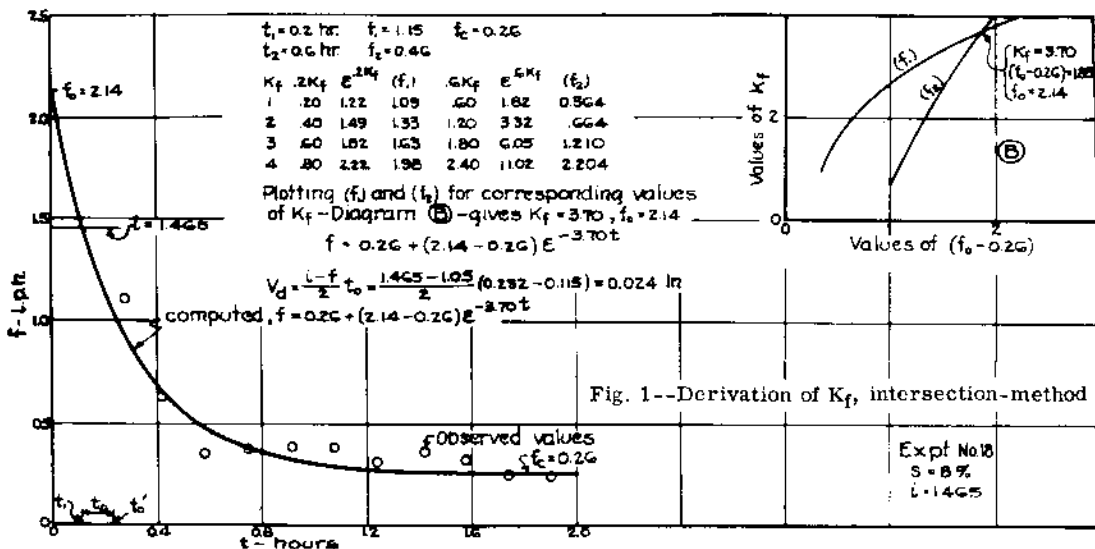The values of f_0 and f_c are derived directly from the infiltration-capacity curve.

Fig. 1--Derivation of K_f, intersection-method

Heretofore two methods have been developed for determining K_f, namely, the method of intersections and the logarithmic method [3]. Figure 1 is an example of an f-curve and its equation derived by the intersection-method. Both these methods of determining K_f, while accurate, are somewhat involved and laborious.

A third method, that of taking the approximate value of t_c directly from the f-curve and then computing K_f from equation (3) derived from (2),

$$K_f = (1/t_c) \log_e [100 (f_0 - f_c)/f_c]$$ (3)

has been tried but is not recommended. The value of t_c cannot be determined with sufficient accuracy by direct inspection from the f-curve. A large personal equation may be involved in its determination, and values of K_f derived in this way by two independent workers may easily differ so much as to give widely different f-curves.

The value of K_f so determined can be checked by computing the corresponding f-curve and comparing it with the original, but to secure agreement therewith may require the use of two or three trial values of f_c, and so the labor of determining K_f becomes even greater than where the method of intersections or the logarithmic method is used.

The following method of deriving K_f from an f-curve is much simpler than any of the three methods described, and gives results of a high order of accuracy.

The total or mass-infiltration, in inches depth, from the beginning of rain (t = 0) to the time t is

$$F = \int_0^t f\, dt$$ (4)

Substituting the value of f from (1) and integrating gives

$$F = f_c t + [(f_0 - f_c)/K_f] [1 - e^{-K_f t}]$$ (5)

The second term in the right-hand member of (5) represents the total mass-infiltration in time t taking place at rates in excess of f_c or, referring to Figure 2, it represents the area between the f-curve, abc, and the asymptotic line, od, for any time-interval t. Taking t equal to infinity, the

Fig. 2--Determination of F_c
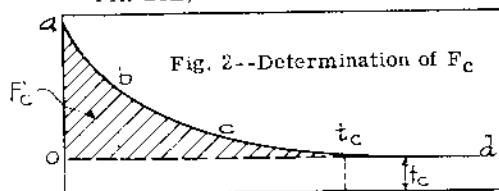
right-hand term of (5) reduces to

$$F_c = [(f_0 - f_c)/K_f] \quad \text{and} \quad K_f = [(f_0 - f_c)/F_c] \tag{6}$$

This quantity F_c does not differ appreciably from the area oabc from $t = 0$ to $t = t_c$, and this area can be determined accurately from the f-curve by averaging the ordinates between zero and t_c or by planimeter-measurement. Hence the value of F_c is known and K_f can be determined at once by substituting this value of F_c in (6). Using the example shown on Figure 1, the value of F_c measured from the curve is 0.506 inch, $f_0 = 2.14$ iph, $f_c = 0.26$ iph, $(f_0 - f_c) = 1.88$ iph. From (6) this gives $K_f = 3.72$ as compared with 3.70 derived by the intersection-method.

In another example, with widely different conditions and from a different series of experiments, F_c derived from the curve = 0.2145, $f_0 = 2.045$, and $f_c = 0.145$. This gives $K_f = 8.88$, which is in good agreement with the value 8.75 derived by the intersection-method.

References

[1] ROBERT E. HORTON, Analysis of runoff-plat experiments with varying infiltration-capacity, Trans. Amer. Geophys. Union, 1939, pp. 693-711.

[2] ROBERT E. HORTON, An approach toward a physical interpretation of infiltration-capacity, Proc. Soil Sci. Soc. Amer., v. 5, pp. 399-417, 1940.

[3] ROBERT E. HORTON, Derivation of infiltration-capacity curve from infiltrometer-experiments, U. S. Dept. Agric., Off. Land Use Coord., Flood Control, Washington, D. C., Mimeo., 18 pp., January, 1942.

Voorheesville, New York

DISCUSSION

H. B. INGERSOLL (Soil Conservation Service, Washington, D. C.; communicated)--The paper presents a direct and useful method of determining the value of K_f. The other constants, f_0 and f_c, may be determined as described by the author in his analysis of NEAL'S experiment [author's reference 1].

These constants apply to the infiltration-capacity curve; not to the so-called $(i - q)$-curve, or to the $(i - q')$-curve, wherein the $(i - q)$-curve is corrected for surface-detention. To derive infiltration-capacity curves, estimates must be made for the values of surface-detention, depression-storage, and interception. Then, to calculate surface-runoff from an infiltration-capacity curve, it is necessary to use these values again as they are not reflected in the infiltration-capacity curve.

Where infiltration-capacity curves are being used in surface-runoff evaluation, values of K_f, f_0, and f_c are required to permit a rational means of averaging or compositing several observations. It is sometimes assumed that the composite infiltration-capacity curve, derived from several "runs" under the same conditions, is obtained more readily by compositing the infiltration-constants than by mere averaging of ordinates of various curves for individual "runs". But where surface-runoff is being estimated by "analogy", that is, by assuming that the hydrograph for an average "run" under given conditions is analogous in every respect to that for a typical unit of area, it is sometimes considered unnecessary to derive the infiltration-capacity curve. For this method surface-detention as observed is modified to appear at its relative time of generation, and the resulting curve, rather than the infiltration-capacity curve, is used. Such curves are usually different for different rainfall-intensities, but the infiltration-capacity curve is usually considered relatively constant, hence values of K_f, f_0, and f_c are real reflections of basic constants. On the other hand, these constants cannot be derived without estimating values of depression-storage, interception, and possibly other phenomena which in themselves are volumetric--and therefore depend on rainfall-intensity. For this reason, the ultimate effect of variations in rainfall-intensity must be considered, no matter which type of curve is used, before surface-runoff can be calculated from infiltrometer-experiments.