It was recommended on the basis of these data to increase by 0.5 freeboard selected for the earth-dam and to strengthen the revetment of the dam-slope. It was further recommended to build harbors at the reach near the mouth of the above-mentioned tributaries in order that ships might find refuge there during storms.

The work belonging to the second point of the program consisted in accumulation of data from technical literature (from 22 Russian and four foreign sources) concerning theoretical and empirical investigations of waves, with the purpose of using these data in the work denoted under point (4) in the program. A short report summarizing the data accumulated has been compiled. The work belonging to the third point of the program consisted in arrangement of field-data obtained at the Onega, Vyg, and other lakes in 1931 and 1932. A detailed report of that work was compiled containing about 250 pages of text and tables and 45 drawings. These data comprise the elements of wind-waves forming in deep water, the transformation of waves on shoals, and the phenomena of wave breaking at the shore line. They have been obtained under various physical and geographical conditions on five lakes, for wind-velocities from four to more than 20 meters per second, for the range of wind-action from 0.4 km to 30 km, and for depth from 0 to 17 meters.

The number of measurements and the amplitude of their values are tabulated below.

Elements of the wave	Number of observations	Amplitude of measurements
Height of wave, h, in meters	2246	0.05 - 3.20
Rise of wave-crest above the middle plane, a, in meters	2246	0.05 - 1.90
Length of wave, L, in meters	226	3.8 -23.6
Period of wave, T, in seconds	2564	1.6 - 5.7
Velocity of propagation, v, in meters per second	226	1.4 - 6.9
Ratio of height to length	226	0.024- 0.1
Ratio of depth of water to length of wave	226	0.05 - 2.18

In addition to these, data were obtained concerning the action of waves upon the shores, the change of the direction of wave-propagation with the approach to shores, and the wind and water-levels regime during storm weather.

The data obtained represent a sufficiently reliable basis for the development of practical formulas expressing the relations between various wave-elements and external factors upon which these elements depend (velocity and duration of wind, range of wind-action). Such formulas would be applicable, of course, for conditions existing on inland bodies of water, such as lakes and impounded rivers.

The work belonging to the fourth point of the program consisted in an analysis of existing formulas and in developing new ones for the following items:

(1) The height and length of wave, as depending upon the range of wind-action and upon the velocity and duration of wind.

(2) The transformation of waves while passing through shallow places.

(3) The determination of depths at which a wave is broken (depending also upon the height of wave and other factors).

It may be noted that the Stevenson's formula commonly used in engineering practice (deduced for conditions of sea-bays) and containing an expression of wave-height in function of the range of wind-action, gives values which are below actual results (this refers to lakes and artificial reservoirs). That formula, therefore, must be replaced by a new one, giving more accurate results.

Carnegie Institute of Technology,
Pittsburgh, Pennsylvania

APPENDIX D--RAIN WAVE-TRAINS

Robert E. Horton

<u>Introduction</u>--Water is sometimes observed to flow in a shallow channel not as uniform flow but as a train of uniformly spaced, traveling, standing waves, sometimes called roll-waves. The hydraulics of such wave-trains are not well determined. Similar trains of waves may sometimes be observed on smooth slopes as a direct result of intense rain. In this case the volume of flow is not uniform but increases progressively down-slope. Such a series of waves resulting from rain may be designated a <u>rain wave-train</u>. The author has carefully observed (but not measured) such rain wave-trains on two occasions, both in intense rains, one on the concrete slope of Loudenville Reservoir, Albany, New York, on a slope of 1 1/2 on 1. The other was on a moderately sloped paved alley in Richmond, Virginia. Rain wave-trains may, therefore, occur on either moderate or steep slopes. The spacing of the waves was apparently about three feet in the first and six feet in the second instance, and the velocity was much higher on the sleep slope. In both cases the waves appeared to remain nearly uniformly spaced from the head to the

foot of the slope although their volumes increased proceeding down-slope. The author has not observed and no one seems to have published an account of rain wave-trains on natural ground-surfaces.

It appears that rain wave-trains may be productive of greatly increased soil-erosion as compared with overland flow without waves. It is probable that rain wave-trains account for some otherwise obscure phenomena not uncommon in cloudburst-floods. These include instances of the tearing up of the sod in strips down a hill-slope and the origin of mud-flows. These phenomena will be more fully treated in another paper [see 1 of "References" at end of paper]. Most persons think of mud-flows as originating in stream-channels. There is evidence that debris carried in mud-flows, such as fences, livestock, and even large boulders, are derived not from the stream-channels but from adjacent hillsides. It is difficult to account for this on the basis of overland flow without waves, since the surface-depth on steep slopes where this phenomenon occurs is rarely more than one inch even in the most intense rains. For overland flow without waves, surface-runoff begins gradually and increases at a more or less uniform rate. Debris brought down by a stream, whether derived from the stream-channel or from the adjacent hillsides, would therefore be expected to come not at or near the front of the rise but later on and in increasing volume. The explanation usually given is that the first debris obstructs the flow and allows more debris to accumulate until a mud- and debris-front is built up in the stream-channel. Suppose, on the other hand, that debris from the hill-slopes begins to enter the channel abruptly and in large volumes early in the stream-rise--all the subsequent phenomena of the mud-flow would be readily understandable. The observed phenomena seem to require an abrupt change in the type of overland flow at about the time the overland flow-profile becomes stable and surface-runoff becomes noticeably intense. This change in type of overland flow must be such that the depth and erosive power of overland flow is greatly augmented. These results would follow from an abrupt change of overland flow from the quasi-steady type to the wave-type. The expression "quasi-steady flow" may be used to describe either overland flow or channel-flow increasing uniformly in volume proceeding downstream or down-slope and flowing without abrupt changes of depth or velocity.

In the following paragraphs an attempt is made to determine the conditions under which rain wave-trains may form on steep slopes.

Conditions of formation of rain wave-trains--Harold Jeffreys [2] has studied mathematically the conditions of occurrence of wave-trains in channels with uniform flow. While some of his hydraulic assumptions appear faulty, these probably do not affect the particular results here cited. Jeffreys concludes that instability which may give rise to a wave-train with uniform flow in the channel will result if the relations of depth, slope, and roughness are such that the velocity for steady flow becomes $v \geqq 2\sqrt{g\Delta}$, where Δ is the depth in feet, v is the velocity in feet per second, and g is the acceleration of gravity.

It is well known that a standing-wave may form at a change of channel-section and that a bore or traveling standing-wave may occur as the result of an abrupt influx of water to a channel when the velocity for uniform flow attains Belanger's critical value $v_c = \sqrt{g(A/W)}$, A being the cross-section and W the width of the channel. For a wide channel this reduces to $v_c = \sqrt{g\Delta}$. It appears from Jeffreys' result that the occurrence of a wave-train in a channel requires twice as great an initial velocity as that required for a bore. This, if true, accounts in part for the infrequent occurrence of wave-trains in streams. On the basis of Jeffreys' result, the ratio of velocity to depth for the occurrence of wave-trains must be $v/\Delta = 2\sqrt{g/\Delta}$. In natural stream-channels the slope, depth, and roughness are usually such that requisitely high values of v/Δ are seldom attained within the natural range of variation of Δ.

It can readily be shown that for quasi-steady overland flow resulting from rain, the profile of the overland flow for turbulent flow in terms of distance x, measured down-slope from the divide, is characterized by the following equations [3]. The equation of the profile of overland flow is

$$\Delta_x = (1/12)\ [(\sigma/K_S)(x/\ell_0)]^{3/5} \tag{1}$$

The mean velocity of overland flow at the point x is

$$v_x = 0.0177\ (S^{0.3}/n^{0.6})\ (\sigma x)^{0.4} \tag{2}$$

and the discharge-rate, in inches depth per hour, is

$$q_S = (1020/n\ell_0)\ (12\Delta)^{5/3}. \tag{3}$$

In (1) the value of K_S is $K_S = 1020\sqrt{S}/n\ell_0$, where Δ = depth of sheet flow at x in feet, x = distance in feet from crest of slope, ℓ_0 = length of slope in feet, S = slope, n = coefficient of roughness in the Manning formula, σ = (i - f) where i is the rain-intensity in inches per hour, and f is the infiltration-capacity in inches per hour.

Dividing (2) by (1) gives, for quasi-steady overland flow

$$v/\Delta = (1.486/12^{2/3}n)\ (\sigma x/K_S\ell_0)^{1/5}. \tag{4}$$

This shows that the ratio v/Δ increases slowly as the distance x, measured down-slope, increases. Slopes on steep areas where cloudburst-floods produce great erosional damage are in general relatively short. A rain wave-train must, therefore, form at not too great a distance from the crest of the slope in order to produce cloudburst-flood effects. The distance x_c from the crest of a slope at which a rain wave-train may form can be obtained by equating the velocity of overland flow, as given by (2), with the critical velocity. Inasmuch as there is, as subsequently appears, some question as to whether the condition precedent for wave-trains is a velocity $2\sqrt{g\Delta}$, as given by Jeffreys, or $\sqrt{g\Delta}$, in accordance with the theory of bores, determinations of the value of x_c or distance from the crest of slope at which rain wave-trains may form, have been computed on both bases. On Jeffreys' basis, with $v_c = 2\sqrt{g\Delta}$,

$$v_x = 0.0177\ (S^{0.3}/n^{0.6})\ (\sigma x)^{0.4} = 2\sqrt{g/12}\ (\sigma x/K_S\ell_0)^{0.3}$$

Solving for x

$$0.00885\sqrt{12/g}\ (\sigma x)^{0.1}\ (K_S\ell_0)^{0.3}\ S^{0.3}/n^{0.5} = 1$$

$$x = (23.1487)^{10}\ (1/\sigma)\ (n^9/S^{4.5}) \tag{5}$$

or

$$x = 44{,}177{,}000{,}000{,}000\ n^9/\sigma S^{4.5} \tag{6}$$

On the basis of Belanger's critical velocity

$$v_c = \sqrt{g\Delta}$$

$$v_x = 0.0177\ (S^{0.3}/n^{0.6})\ (\sigma x)^{0.4} = \sqrt{g/12}\ (\sigma x/K_S\ell_0)^{0.3}$$

Solving for x

$$0.0177\ \sqrt{12/g}\ (\sigma x)^{0.1}\ (K_S\ell_0)^{0.3}\ S^{0.3}/n^{0.6} = 1$$

$$x = (11.58)^{10}\ (1/\sigma)\ (n^9/S^{4.5}) \tag{7}$$

$$= 43{,}360{,}000{,}000\ n^9/\sigma S^{4.5} \tag{8}$$

Computations of x_c have been made for a rough surface, with σ = 1.0 and n = 0.050, with results as given in Table 1. This shows at once that for a rain wave-train to form close to the crest of the slope on a natural ground-survace, the slope must be steep. Thirty per cent or more is required on the bore-theory, whereas on Jeffreys' basis a slope of 100 per cent or more would be required. Similar computations for a smooth slope, on Jeffreys' basis, show that with n = 0.010, a rain wave-train to form close to the crest of the slope requires a slope of 20 per cent or more, whereas on the bore-basis, a rain wave-train may form close to the head of any slope exceeding one or two per cent. Inasmuch as the author has observed well-formed rain wave-trains in a paved alley at Richmond, Virginia, with a slope not exceeding one or two per cent, the question may fairly be raised whether Jeffreys' criterion is correct in the case of rain wave-trains.

Table 1--Values of x_c for σ = 1.0

S	Using $2\sqrt{g\Delta}$ x_c	Using $\sqrt{g\Delta}$ x_c
.10	2,610,000	2567.
.20	120,000	119.
.30	19,400	19.1
.40	5,340	5.25
.50	1,950	1.92
.60	862	0.844
.80	235	0.231
1.00	86	0.085

Crest-belt of laminar flow--Since laminar flow does not produce surface-erosion, the distance down-slope to which the belt of laminar flow extends is of interest. The minimum velocity of overland flow for which the flow can be laminar is given by the author's criterion [4]

$$v_H = \nu/5n^2\ \Delta^{2/3} \tag{9}$$

where ν is the kinematic viscosity of water in foot-pound-second units. Equating this with the velocity of overland flow, (2),

$$\nu/5n^2\ \Delta^{2/3} = 0.0177\ (S^{0.3}/n^{0.6})\ (\sigma x)^{0.4}$$

where Δ is the surface detention-depth in feet.

Substituting the value of Δ from (3)

$$\Delta_x = \frac{1}{12}\left(\frac{\sigma x}{K_S \ell_0}\right)^{3/5}$$

$$\nu = 5n^2\left[\left(\frac{1}{12}\right)^{2/3}\left(\frac{\sigma x}{K_S \ell_0}\right)^{2/5}\left(0.0177\ \frac{S^{3/10}}{n^{3/5}}\ (\sigma x)^{2/5}\right)\right]$$

$$= \frac{0.0885\ n^{7/5}}{(12)^{2/3}}\ \frac{(\sigma x)^{4/5}\ S^{3/10}}{(K_S \ell_0)^{2/5}}$$

and solving for x

$$x = \frac{1}{\sigma}\left[\frac{\nu (12)^{2/3}\ (K_S \ell_0)^{2/5}}{(0.0885)\ (n)^{7/5}\ S^{3/10}}\right]^{5/4} = \frac{\sqrt{K_S}\ \sqrt{\ell_0}\,\nu}{\sigma n^{7/4}\ S^{3/8}}\left(\frac{12^{5/6}}{0.0885^{5/4}}\right)$$

Since $K_S = 1020\sqrt{S}/n\ell_0$

$$x = \frac{\sqrt{1020}\ S^{1/4}\ \sqrt{\ell_0}\,\nu}{\sqrt{n}\ \sqrt{\ell_0}\,\sigma n^{7/4}\ S^{3/8}}\left(\frac{12^{5/6}}{0.0885^{5/4}}\right) = \frac{524\,\nu}{\sigma n^{9/4}\ S^{1/8}} \tag{10}$$

For 60° F, $\nu = 0.0000121$, $x = 0.00634/\sigma n^{9/4}\ S^{1/8}$. With $\sigma = 1$ and $n = 0.05$,

$$x = 5.373/S^{1/8} \tag{11}$$

Table 2--Values from equation (11)

S	$S^{1/8}$	x_t	S	$S^{1/8}$	x_t
0.01	0.561	9.56	0.50	0.915	5.87
0.10	0.750	7.16	0.60	0.938	5.73
0.20	0.817	6.58	0.80	0.973	5.53
0.30	0.859	6.25	1.00	1.00	5.37
0.40	0.891	6.03			

The width of the belt of laminar flow at the head of a slope does not exceed 10 feet for $\sigma = 1.0$ for slopes ≧ one per cent and is less for larger values of σ.

Laminar flow does not inhibit the formation of rain wave-trains on rough slopes. On a smooth slope, with n = 0.01, the value of x_t is $5^{9/4}$ or 37.4 times as great as for n = 0.05, and a rain wave-train can not extend close to the head of the slope unless σ is large. On smooth slopes, where rain wave-trains are most likely to be observed, their occurrence is likely to be inhibited by laminar flow except for extremely high values of σ.

Erosional power of rain wave-trains--Figure 1 shows, schematically, (a) a profile of quasi-steady overland flow and (b) the same flow broken up with a wave-train.

Using DuBoys' formula for the tractive force per unit of boundary-surface with which a flowing stream is in contact, and introducing a factor C_b equal to the ratio of bottom to mean velocity, to correct for the fact that the tractive force is exerted at a rate $C_b v$--not at the mean velocity--gives

$$F = w_1 \Delta S/C_b \tag{12}$$

where F = tractive force in pounds per square foot, w = weight of water and suspended sediment in pounds per cubic foot, Δ = depth of overland flow or average height of wave in feet, S = slope. For turbulent flow, except on a sod-surface, C_b is commonly 0.30 to 0.60; its average is about 0.50. For uniform overland flow produced by rain on slopes, Δ is usually not over 0.1 foot.

It will be seen from (1) that the profile of overland flow for turbulent flow is a parabola

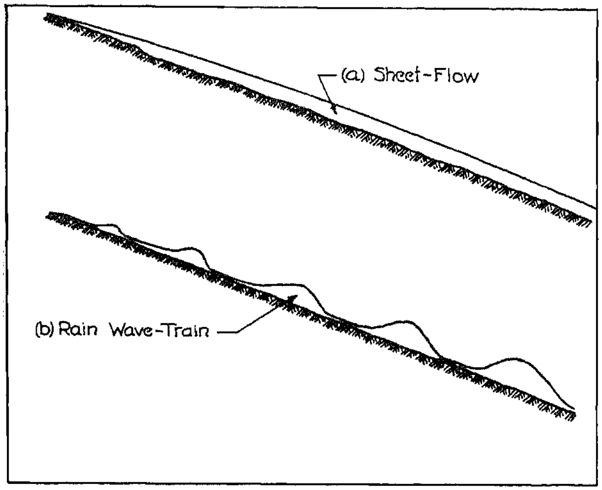

FIG. 1—SHEET-FLOW AND RAIN WAVE-TRAIN (Schematic)

with exponent 3/4 and that the average depth of surface-detention on the slope upstream from x is $3/8\Delta_x$. If, instead of uniform flow, a rain wave-train occurs with waves spaced at an interval z, then each succeeding wave sweeps into itself the accumulated surface-detention since the passage of the last preceding wave. When a wave-train occurs there will be ℓ_0/z waves on the slope, and the average volume per wave will be

$$V_w = (3/8)\Delta z \qquad (13)$$

In the rain wave-trains observed, the waves had base lengths of perhaps one-half foot at the head of the slope, increasing to one or one and one-half feet at the foot of the slope, the bases being longer for the waves on the flatter slopes. Calling the average base length per wave b, the average wave depth Δ_w will be

$$\Delta_w = (V_w/b) = (3/8)\ (\Delta z/b) \qquad (14)$$

In the cases observed, the average value of b was about (1/4)z to (1/6)z. Taking the average value of b from the top to the foot of the slope as z/5, gives the relation

$$\Delta_w/(3/8)\Delta = z/b = \text{say } 5.0 \qquad (15)$$

This is the ratio of the average wave-depth to the average depth of surface-detention on the slope, the latter for flow without a wave-train. Since tractive force is proportional to the depth, and erosional power is a function of tractive force and probably proportional to it, it appears that the erosional effect of a rain wave-train on the soil-surface may be of the order five times the erosional effect of the same volume of overland flow without waves. The difference is that between a steady moderate pull and a succession of sharp blows repeated at uniform intervals.

The preceding analysis throws light on other characteristics of rain wave-trains, particularly their apparently nearly constant velocity proceeding down the slope. The volume of the wave increases in proportion to the distance x travelled. If the velocity remains constant, obviously the wave-spacing z will remain constant. The velocity can, therefore, remain constant only in case the frictional resistance increases in proportion to the wave-volume. The wave-sections are apparently nearly parabolic and the average wave-depth about two-thirds the crest depth, so that the area of wave-section at a given point x is $A = (2/3)\Delta b$. From DuBoys' law, the tractive force per unit of base area is proportional to Δ, and the total tractive force per wave is proportional to Δb. The energy available to overcome resistance and produce velocity is proportional to A, and since $A = (2/3)\Delta b$, it is evident that the velocity may remain sensibly constant and the waves equally spaced as they travel down the slope.

Velocity of wave-trains--Considering wave-trains in channels, Jeffreys' analysis as well as observations of Vaughan Cornish on wave-trains in the Grünnbach conduit [5] indicate that where there is an appreciable antecedent depth of water, a wave-train virtually rides the antecedent flow at a velocity $\sqrt{g\Delta}$ in relation thereto. Consequently if the antecedent or initial velocity equals the critical instability-velocity given by Jeffreys, that is, $v_c = 2\sqrt{g\Delta}$, then the wave-velocity relative to the Earth is $w = 3\sqrt{g\Delta}$ and $w/v = 1.5$. Cornish remarks (p. 327): "I have never seen a stream with a uniform depth of more than four inches adopt spontaneously the intermittent flow in a series of roll-waves." Apparently there is a distinction between the channel wave-trains observed by Cornish and discussed by Jeffreys, and rain wave-trains here under consideration. In the case of the former there is an appreciable antecedent depth and the wave-train rides on the antecedent flow at a velocity $\sqrt{g\Delta}$.

Equating $v_c = 2\sqrt{g\Delta}$, as given by Jeffreys, with the velocity as given by the Manning formula

$$v = 2\sqrt{g\Delta} = 11.33\sqrt{\Delta} = (1.486/n)\,\Delta^{2/3}\sqrt{S}$$

$$11.33\sqrt{\Delta}\,n = 1.486\Delta^{2/3}\sqrt{S}$$

$$11.33n/1.486\sqrt{S} = \Delta^{1/6}$$

$\Delta_c = (7.62)^6\,n^6/S^3$ = least depth at which a wave-train can occur.

$$\Delta_c = 195{,}750\,n^6/S^3 \quad (16)$$

By a similar analysis for $v_c = \sqrt{g\Delta}$

$$\Delta_c = 3215\,n^6/S^3 \quad (17)$$

The value of Δ_c comes out extremely small. Computed values are given in Table 3.

Table 3--Computed values of Δ_c from equation (17)

S	S^3	n = 0.010 Δ_c	n = 0.050 Δ_c
0.10	0.001	0.00019570	3.059
0.25	0.0156	0.00001252	0.1961
0.50	0.125	0.000001564	0.02446
1.00	1.0	0.00000019570	0.003059
1.50	3.375	0.0000000579	0.000906
2.00	8.00	0.00000002446	0.0003824

The value of Δ_c varies inversely as S^3. It is to be kept in mind that slopes tributary to a stream are always steeper than the stream-channel. For some mountain streams in the State of New York the author found the slope-ratio (S_g/S_c) of the ground to the stream-channel to be of the order 5.0 [6].

The author's observations and preceding calculations indicate that in the case of rain wave-trains the antecedent depth is negligible and the succeeding waves sweep into their volume the antecedent flow and virtually glide down the slope, apparently at a velocity $\sqrt{g\Delta}$, as in the case of bores. All the facts point strongly to the conclusion that, while the conditions precedent for wave-trains in a channel of appreciable depth is probably $v_c = 2\sqrt{g\Delta}$, as given by Jeffreys, rain wave-trains may form with $v_c = \sqrt{g\Delta}$.

Effect of grass on rain wave-trains--As elsewhere shown by the author [7], a dense cover of grass subdivides overland flow in such a manner that the hydraulic radius and consequently the velocity of overland flow is sensibly constant not only at different depths at the same point but at different points down a stream-slope. Under these conditions the ratio v/Δ decreases with distance x, measured down-slope, and wave-trains are apparently impossible on well-sodded surfaces as long as the grass remains standing.

In case of cloudbursts, where the sod has been torn up, the author's observations indicate that the grass has first been beaten down flat on the ground. Since the retardation of overland flow by grass greatly increases the surface-detention, if the grass is beaten down, a considerable volume of surface-detention is more or less abruptly released. Under these conditions this may lead to the abrupt formation of a rain wave-train, with greatly increased intensity. Grass has the same effect on flow as greatly increased roughness. This cuts down the width of the belt of laminar flow. Other things permitting, rain wave-trains can form close to the crest of a sodded slope. Grass and roughness projections on the soil-surface tend, however, to throw off reflected waves and disperse a rain wave-train in much the same way that irregularities of a channel-section tend to disperse wave-trains in channels. This may account for the absence of observations of rain wave-trains on natural ground-slopes. Cornish has called attention to a similar effect in channel wave-trains, where the wave-trains would not be noticed unless looked for in the confused water surface-conditions.

Conclusions--It is interesting to note that recent activities in the study of soil-erosion have brought to light some new things in hydraulics, including the possibility of the occurrence of subdivided sheet-flow, with sensibly constant velocities on sod-covered slopes, and the possibility of occurrence of rain wave-trains on steep slopes. This paper is intended merely as an introduction to the subject of rain wave-trains, a subject which has apparently nowhere received attention hitherto in hydraulic and hydrologic literature.

The preceding study of rain wave-trains is based on mathematical analyses and ocular observations. It leads to results in accordance with the available data. The following conclusions should, however, be checked experimentally:

(1) The occurrence of rain wave-trains on natural ground surfaces is limited to steep slopes and high rain-intensities.

(2) The waves which form a train may have a tractive force or eroding power of the order of five times that of the same flow without waves.

(3) Standing grass inhibits the formation of rain wave-trains by preventing the instability-ratio v/Δ to attain sufficiently requisite values, but grass beaten down by rain or overland flow may accentuate wave-train erosion thereafter by releasing excess surface-detention.

(4) Wave-trains are shallow-flow phenomena and are inhibited in most natural channels by the fact that the depth is never sufficiently small to give the requisite value of v/Δ for instability.

(5) The waves in a rain wave-train apparently travel with a constant velocity $\sqrt{g\Delta}$ and remain spaced at nearly uniform intervals down the slope but increase in depth, volume, and width of base as they proceed down-slope.

References

[1] Robert E. Horton, Cloudburst-rains and cloudburst-floods.
[2] Harold Jeffreys, The flow of water in an inclined channel of rectangular section, Phil. Mag., ser. 6, v. 49, pp. 793-803.
[3] Robert E. Horton, Surface-runoff phenomena, part I: Analysis of the hydrograph, Pub. 101, Horton Hydrologic Laboratory, Voorheesville, New York, 1935.
[4] Horton, Leach, and Van Vliet, Laminar sheet flow, Trans. Amer. Geophys. Union, 1934, pp. 394-404.
[5] Vaughan Cornish, Waves of the sea and other water waves, Open Court Pub. Co., Chicago, Illinois.
[6] Robert E. Horton, Drainage-basin characteristics, Trans. Amer. Geophys. Union, 1932, pp. 350-361.
[7] Robert E. Horton, Surface-runoff control; Headwaters control and use, Govt. Printing Office, Washington, 1937, pp. 16-41.

Voorheesville, New York

APPENDIX E--SEDDON'S AND FORCHHEIMER'S FORMULAS FOR CREST-VELOCITY OF FLOOD-WAVES SUBJECT TO CHANNEL-FRICTION CONTROL

Robert E. Horton

Introduction and synopsis--Among the least-understood phenomena of runoff are those involving the time of transit of the crest of a stream-rise from a given point of channel-inflow to an outlet. Practical importance attaches to this question in connection with flood-stage prediction. Time of transit is also involved where attempts are made to determine channel-outflow by the time-contour method. The validity of that method depends to a considerable extent on the accuracy with which time of transit can be determined.

There is a growing belief that the time of transit of a stream rise is governed by some type of wave-velocity and is not governed by the velocity of ordinary hydraulic flow--in other words, that the wave-crest velocity and the velocity at which water would flow at crest-depth, in accordance with, say, the Manning formula, are not the same, and that the former applies more nearly to the movement of stream-rises.

It seems obvious, although it does not appear to have been clearly pointed out before, that in natural stream-channels there is a range of conditions extending from those in which the wave-movement is almost completely subject to channel-friction control, to those in which the wave-movement is almost exclusively subject to gravity- or momentum-control, and channel-friction plays only a negligible role.

The conditions to which these two cases apply are apparently related through three factors: (1) Wave-length relative to wave-height; (2) wave-height relative to initial depth; (3) wave-height relative to hydraulic radius or, perhaps better, to the reciprocal of the hydraulic radius. Hydraulic considerations, as well as the author's experiments [see 1 of "References at end of paper], indicate clearly that the movement of channel-waves changes gradually from that subject to momentum-control to that subject to friction-control as the ratio of wave-length to wave-height increases. While apparently the type of wave-control, whether by momentum or friction, is also related to the last two factors, the nature of this relationship is as yet undetermined. As the flood-wave due to a stream-rise traverses the system of stream-channels, the